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CONTROL COORDINATION OF LARGE SCALE HEREDITARY SYSTEMS

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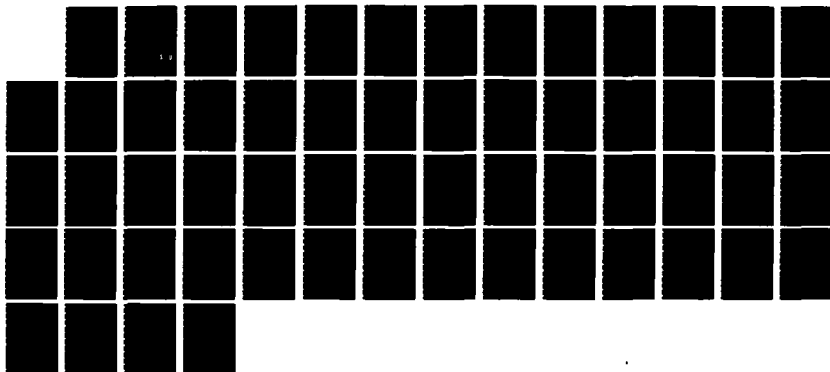
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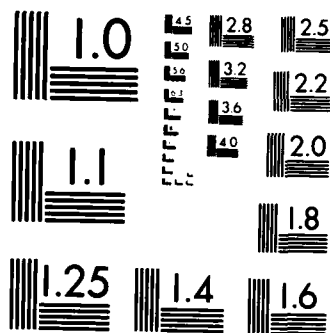
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REPORT DOCUMENTATION PAGE

1a. REPORT SECURITY CLASSIFICATION UNCLASSIFIED			1b. RESTRICTIVE MARKINGS		
2a. SECURITY CLASSIFICATION AUTHORITY ---			3. DISTRIBUTION/AVAILABILITY OF REPORT Approved for public release; distribution unlimited		
2b. DECLASSIFICATION/DOWNGRADING SCHEDULE N/A			5. MONITORING ORGANIZATION REPORT NUMBER(S) AFOSR AFOSR-TR- 86-0895		
4. PERFORMING ORGANIZATION REPORT NUMBER(S)			7a. NAME OF MONITORING ORGANIZATION AFOSR		
6a. NAME OF PERFORMING ORGANIZATION Clemson University		6b. OFFICE SYMBOL (If applicable)	7b. ADDRESS (City, State and ZIP Code) Building 410 Bolling AFB DC 20332-6448		
6c. ADDRESS (City, State and ZIP Code) Mathematical Sciences Clemson SC 29631		8b. OFFICE SYMBOL (If applicable) NM			
8a. NAME OF FUNDING/SPONSORING ORGANIZATION AFOSR		9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER AFOSR 84-0236			
8c. ADDRESS (City, State and ZIP Code) Building 410 Bolling AFB DC 20332-6448		10. SOURCE OF FUNDING NOS.			
		PROGRAM ELEMENT NO. 61102F	PROJECT NO. 2304	TASK NO. A1	WORK UNIT NO.
11. TITLE (Include Security Classification) Coordination of Large Scale Hereditary Systems					
12. PERSONAL AUTHOR(S) Robert Fennell					
13a. TYPE OF REPORT ANNUAL		13b. TIME COVERED FROM 71/84 TO 711/85		14. DATE OF REPORT (Yr., Mo., Day)	
				15. PAGE COUNT 55	
16. SUPPLEMENTARY NOTATION					
17. COSATI CODES			18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)		
FIELD	GROUP	SUB. GR.			
19. ABSTRACT (Continue on reverse if necessary and identify by block number)					
OVER					
<div style="text-align: right;"> DTIC ELECTE OCT 08 1986 S D </div>					
<div style="text-align: left;"> DTIC FILE COPY </div>					
20. DISTRIBUTION/AVAILABILITY OF ABSTRACT UNCLASSIFIED/UNLIMITED <input type="checkbox"/> SAME AS RPT. <input checked="" type="checkbox"/> DTIC USERS <input type="checkbox"/>			21. ABSTRACT SECURITY CLASSIFICATION UNCLASSIFIED		
22a. NAME OF RESPONSIBLE INDIVIDUAL JOHN P. THOMAS, CAPT, USAF		22b. TELEPHONE NUMBER (Include Area Code) 202/767-5026		22c. OFFICE SYMBOL NM	

A decentralized control strategy, which allows some level of autonomous component control for large scale hereditary systems, has been developed. The basic problem considered is how to add constraints to the component objectives and to arrange exchanges of information which enable all components to achieve their objectives. The approach requires that component interactions be suitably limited so that variational methods can be used to determine component controls independently. A reproducing kernel Hilbert space of Hellinger integrable functions provides the setting for the description of system operators and the analysis of optimization problems. This work has been reported in papers entitled: "Decentralized Control for Large Scale Hereditary Systems" and "Control Coordination for Large Scale Systems." In other related work geometric, algebraic, and graph theoretic properties of system matrices have been characterized which allow some autonomy in the choice of component control laws. This is reported in the paper entitled "Canonical Forms for Decentralized Control."



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GRANT AFOSR-84-0236

CONTROL COORDINATION FOR HEREDITARY SYSTEMS

INTERIM REPORT

July 1, 1984 - July 1, 1985

Decentralized Control. A decentralized control strategy, which allows some level of autonomous component control for large scale hereditary systems, has been developed. The basic problem considered is how to add constraints to the component objectives and to arrange exchanges of information which enable all components to achieve their objectives. The approach requires that component interactions be suitably limited so that variational methods can be used to determine component controls independently. A reproducing kernel Hilbert space of Hellinger integrable functions provides the setting for the description of system operators and the analysis of optimization problems. See the attached papers "Decentralized control for large scale hereditary systems" and "Control coordination for large scale systems" for details.

In related work geometric, algebraic, and graph theoretic properties of system matrices have been characterized which allow some autonomy in the choice of component control laws. See the attached paper "Canonical forms for decentralized control" for details.

Stochastic Systems. Reproducing kernel Hilbert space methods have produced useful approximations for estimation and control problems of deterministic linear hereditary systems. By obtaining explicit reproducing kernel Hilbert space representations of stochastic processes governed by linear hereditary dynamics as spaces of Hellinger integrable functions such approximation methods apply to the problem of finding the covariance kernel given the model. These results appear in the attached paper "RKH space methods for approximating the covariance kernels of a class of stochastic linear hereditary systems, II".

Activities partially supported by Grant AFOSR-84-0236

- 1) Both principal investigators attended the 1984 Mathematical Sciences Lecture Series, Inference for stochastic processes of semi-martingale type, John Hopkins University, July 16-20, 1984.
- 2) Both principal investigators attended the AMS-IMS-SIAM Summer Research Conference in the Mathematical Sciences, Linear Algebra and its Role in Systems Theory, Bowdoin College, Brunswick, Main, July 29-August 4, 1984.
- 3) Both principal investigators attended the Southeastern Atlantic Regional Conference on Differential Equations, Winston Salem, North Carolina, October 12-13, 1984.
 - a) R. Fennell gave a contributed paper "Control Coordination".
 - b) J. Reneke gave a contributed paper "A 'checkable' condition for controllability of linear hereditary systems.

- 4) R. Fennell gave a contributed paper "Control coordination for large scale linear systems" at the SIAM Conference on Applied Linear Algebra, Raleigh, North Carolina, April 29-May 2, 1985.
- 5) J. Reneke gave a contributed paper "RKH space methods for approximating the covariance kernels of a class of stochastic linear hereditary systems, II" (with R. Minton) at the 7th International Symposium on the Mathematical Theory of Networks and Systems, Stockholm, Sweden, June 10-14, 1985.
- 6) R. Fennell gave a contributed paper "Control coordination for large scale systems" (with J.A. Reneke and S.B. Black) at the 1985 American Control Conference, Boston, Massachusetts, June 19-21, 1985.

RESEARCH Papers*

Decentralized Control for Large Scale Hereditary Systems

by J.A. Reneke and R.E. Fennell

Status: Submitted for publication.

: Accepted for presentation at the 1985 IEEE Conference on Decision and Control, Fort Lauderdale, Florida, Dec. 11-13.

Control Coordination for Large Scale Systems

by R.E. Fennell, J.A. Reneke, and S.B. Black

Status: Appeared - Proceedings of the 1985 American Control Conference June 19-21, 1985.

RHK Space Methods for Approximating the Covariance Kernels of a Class of Stochastic Linear Hereditary Systems, II.

by J.A. Reneke and R.B. Minton.

Status: To appear - Proceedings Seventh International Symposium on the Mathematical Theory of Networks and Systems, June 10-14, 1985.

Cononical Forms for Decentralized Control

by R.E. Fennell

Status: Submitted for publication.

*Papers Attached

DECENTRALIZED CONTROL FOR LARGE SCALE HEREDITARY SYSTEMS*

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Abstract: A control strategy, which allows some level of autonomous component control, for large scale hereditary systems is presented. The basic problem considered is how to add conditions to the component objectives and to arrange exchanges of information which enable all components to achieve their objectives. The approach requires that component interactions be suitably limited so that variational methods can be used to determine component controls independently. A reproducing kernel Hilbert space of Hellinger integrable functions provides the setting for the description of system operators and the analysis of optimization problems.

* This research was supported in part by the Air Force Office of Scientific Research under Grant AFOSR-84-0236.

Keywords: Decentralized control, hereditary systems.

Abbreviated Title: Decentralized Control.

DECENTRALIZED CONTROL FOR LARGE SCALE HEREDITARY SYSTEMS

J. A. Reneke and R.E. Fennell

1. Introduction. We present the mathematical theory of a decentralized control strategy which allows some level of autonomous component control for large scale systems. To indicate the difficulties of decentralized control, consider the problem of two controllers independently choosing controls to steer the two component system

$$h'_1(t) = \alpha_{11}h_1(t-1) + \alpha_{12}h_2(t) + \beta_1v_2(t)$$

$$h'_2(t) = \alpha_{22}h_2(t) + \beta_2v_2(t)$$

from a given initial position to a desired terminal position. Although the second component objective can be achieved independent of the first, some information about the second component is necessary for the simultaneous achievement of the first objective. The basic problem considered in this paper, a coordination problem, is how to add conditions to the component objectives and arrange exchanges of information which will enable all of the components to achieve their objectives autonomously. Our approach to the solution of this problem requires that component interactions be suitably limited so that variational methods can be used to determine component control laws.

Current interest in coordination of large scale systems is concentrated on the problem of system optimization for hierarchical or multilevel systems [4], [14], [15]. Methods employed are typically off line iterative and are usually only applied to finite dimensional state space systems. Our methods apply to systems modelled by delay differential equations and integral equations. Also application of these methods to control coordination problems for large scale systems leads to closed loop controls.

In our approach coordination of local control laws is achieved in one step by the imposition of additional constraints on the system components.

Large scale systems will be described by operator equations defined on an infinite dimensional, reproducing kernel Hilbert space and variational methods will be used to solve terminal constraint problems. A similar approach to the solution of optimal control problems for systems modelled by functional differential equations or Volterra integral equations appears in the work of Neustadt [10]. In Section II we present the Hilbert space background necessary for our analysis. Reproducing kernel Hilbert spaces are frequently used for the analysis of system problems, see the texts [3], [16] and the references there. We have previously used a reproducing kernel Hilbert space of Hellinger integrable functions to analyze identification and estimation problems [2], [12]. Such spaces have also arisen in the analysis of stochastic systems [9].

We are interested in terminal constraint and trajectory following problems. In Section III Lagrange Multiplier methods are used to characterize the solution of component control problems with interval constraints. Two local optimization problems are considered. One problem applies to hereditary systems in general while the other applies only to finite dimensional state space systems. In the first problem a novel performance index is used while a more standard quadratic cost functional appears in the second problem.

The ability to coordinate locally designed control laws depends on the type of interaction between system components. The concept of stably connected systems is introduced in Section IV to describe interaction between system components. Coordination strategies can be developed for stably connected systems by adding side conditions to the component

requirements and using the results of the local optimization problems developed in Section III. Finite dimensional state space systems whose system matrix is block triangular or decomposed by strong components [8] are stably connected. In Section III we indicate how coordination strategies can be developed for two component and other multicomponent systems.

2. Preliminaries. For $0 \leq r < T$ let S denote the interval $[-r, T]$, $X = E^d$ denote the space of real d -tuples with the usual inner product $\langle \cdot, \cdot \rangle$ and norm $|\cdot|$. Let G denote the space of functions from S to X which are continuous on S except, possibly, for a jump discontinuity at $t = 0$ (assume right continuity at 0), and let associated semi-norms be defined by $N_x(f) = \sup_{-r \leq t \leq x} |f(t)|$ for x in S and f in G . Let k be a nondecreasing, right continuous function on S with $k(-r) = 1$ and a jump discontinuity at $t = 0$. We define B to be the set of linear transformations of G to which B belongs provided 1) $[Bf](t) = 0$ for $-r \leq t \leq 0$ and 2) there is a constant b such that for any u, v in S with $u \leq v$, $|[Bf](v) - [Bf](u)| \leq b \int_u^v N_x(f) dk(x)$. Operators in B are bounded transformations from $\{G, N_T\}$ to $\{G, N_T\}$. Systems whose dynamics can be described by an operator equation of the form $h = f + Bh$ with B in B are referred to as hereditary systems. The defining properties of the class B not only guarantee the existence and uniqueness of solutions to the systems under consideration but also imply the invertibility of the system input-output operator.

THEOREM 2.1 [12] If B is in B then $I - B$ maps $\{G, N_T\}$ onto $\{G, N_T\}$ and $I - B$ has a bounded inverse.

Thus for B in \mathcal{B} and $A = (I-B)^{-1}$ we have the equivalent system realizations

$$h = f + Bh + \beta u \quad \text{and} \quad h = A(f + \beta u) \quad (2.1)$$

where f, u in G denote system inputs and controls and β denotes a linear transformation from X to X . One should note that $[Af](t) = f(t)$ for each f in G and $-r \leq t \leq 0$.

Example 2.1. As an example consider a system of delay differential equations of the form

$$h'(t) = \alpha_1 h(t) + \alpha_2 h(t-r) + \beta_1 v_1(t) + \beta_2 v_2(t) + e(t)$$

for $0 \leq t \leq T$ and initial condition $h(t) = \phi(t)$ for $-r \leq t \leq 0$. Here $\alpha_1, \alpha_2, \beta_1, \beta_2$ represent matrices of appropriate dimension and r denotes a delay parameter. Taking $[Bh](t) = 0$ for $-r \leq t \leq 0$ and $[Bh](t) = \int_0^t [\alpha_1 h(\tau) + \alpha_2 h(\tau-r)] d\tau$ for $0 \leq t \leq T$, the system may be written as

$$h = f + Bh + \beta_1 u_1 + \beta_2 u_2 \quad (2.2)$$

with $f(t) = \phi(t)$ for $-r \leq t \leq 0$, $f(t) = \phi(0) + \int_0^t e(\tau) d\tau$ for $0 \leq t \leq T$ and $u_i(t) = 0$ for $-r \leq t \leq 0$, $u_i(t) = \int_0^t v_i(\tau) d\tau$ for $0 \leq t \leq T$, $i = 1, 2$. With $k(t) = 1+r+t$ for $-r \leq t \leq 0$ and $k(t) = 2+r+t$ for $0 \leq t \leq T$, it follows that B is in \mathcal{B} . The jump discontinuity in k at $t = 0$ allows a similar discontinuity in the solutions of (2.2).

Analysis of the optimal control problems to be considered is simplified when the domain of operators in \mathcal{B} is restricted to the reproducing kernel Hilbert space G_H of Hellinger integrable functions. Defining properties of this space may be found in the papers [2], [6], [12]. For completeness we mention those properties which will be used in this paper. The space G_H is the subspace of G with inner product denoted by $Q_H(\cdot, \cdot)$,

associated norm $N_H(\cdot)$, and reproducing kernel K defined by $K(s,t)x = k(s)x$ for $-r \leq s \leq t \leq T$, $= k(t)x$ for $-r \leq t \leq s \leq T$ with x in X . Here we restrict k as defined in Example 2.1. The kernel K maps $S \times S$ into the linear transformations of X and satisfies $K(\cdot, t)x$ is in G_H and $\langle f(t), x \rangle = Q_H(f, K(\cdot, t)x)$ for each t in S and x in X . It may be shown that $Q_H(f, g) = \langle f(-r), g(-r) \rangle + \int_{-r}^T \langle df, dg \rangle / dk$ where $\int_{-r}^T \langle df, dg \rangle / dk$ is the limit through refinement of the approximating sums $\sum \langle f(s_i) - f(s_{i-1}), g(s_i) - g(s_{i-1}) \rangle / (k(s_i) - k(s_{i-1}))$ with $\{s_i: i = 0, \dots, n\}$ a partition of S . The following result motivates us to restrict attention to systems described by operator equations on $\{G_H, Q_H\}$.

COROLLARY 2.2 [12] If B is in \mathcal{B} then $I-B$ maps $\{G_H, N_H\}$ onto $\{G_H, N_H\}$ and $A = (I-B)^{-1}$ is a bounded transformation from $\{G_H, N_H\}$ onto $\{G_H, N_H\}$.

For future reference we define on G_H the family of projection operators $\{P_t: t \text{ in } S\}$ by $[P_t f](s) = f(s)$ for $-r \leq s \leq t \leq T$ and $[P_t f](s) = f(t)$ for $-r \leq t \leq s \leq T$. It may be shown that operators B in \mathcal{B} and $A = (I-B)^{-1}$ satisfy the causal property $P_t B = P_t B P_t$, $P_t A = P_t A P_t$ for all t in S . It is also useful to note that $B_1(I-B_2)^{-1}$, $(I-B_2)^{-1}B_1$, and B_1B_2 are in \mathcal{B} whenever B_1, B_2 are in \mathcal{B} . We will make frequent use of the operator C in \mathcal{B} defined by $[Ch](t) = 0$ for $-r \leq t \leq 0$ and $[Ch](t) = \int_0^t h(\tau) dk(\tau)$ for $0 \leq t \leq T$. The adjoint of C is given by $[C^*h](t) = K(t, T)h(T) - K(t, 0)h(0) - [Ch](t)$ since

$$\begin{aligned} Q_H(f, C^*h) &= Q_H(Cf, h) = \int_0^T \langle f, dh \rangle \\ &= \langle f(T), h(T) \rangle - \langle f(0), h(0) \rangle - \int_0^T \langle df, h \rangle \\ &= \langle f(T), h(T) \rangle - \langle f(0), h(0) \rangle - \int_0^T \langle df, dCh \rangle / dk \\ &= Q_H(f, K(\cdot, T)h(T) - K(\cdot, 0)h(0) - Ch) \end{aligned}$$

for all f, h in G_H . Finally, for sufficiently smooth functions note that $N_H^2(f) = |f(0)|^2 + \int_0^T |f'(\tau)|^2 d\tau$ and, hence, $N_H^2(Ch) = \int_0^T h^2(\tau) d\tau$.

We conclude this section by stating a controllability assumption for hereditary systems. Let β be a linear transformation from X to X and let G_U denote the set of u in G_H with the property that $P_0 u = 0$ and $u(t)$ belongs to the orthogonal complement of the null space of β for $-r \leq t \leq T$. Note that G_U is a closed subspace of G_H . We restrict attention to controls u in G_U .

Controllability Assumption: For B in B , and β a linear transformation from X to X assume $A\beta = (I-B)^{-1}\beta$ maps G_U onto $(I-P_0)G_H$.

Note that for $\beta = I$ the invertibility of the input output operator A guarantees controllability. For delay differential equations, (2.1), this assumption implies function space controllability, [1], [7], [13].

3. Local Optimization Problems. The control coordination strategy to be introduced in Section IV requires that component interactions be suitably limited and certain side conditions be added to the component control objective. Local control problems can then be solved without full information of the actions of other component controllers. Lagrange multiplier methods are used to characterize the solution to the local control problems.

In this section, we are concerned with system components whose signal diagram is of the form

Figure 1

Here $A = (I-B)^{-1}$ with B in B ; f in G_H is a known input signal; g in G_H is an unknown input; u in G_U is the component control; and β, γ denote linear transformations from X to X . The signal g represents the effect of other components upon the given component. We assume $P_0 g = 0$ and will add other conditions necessary for the solution of the local optimization problems.

Two optimization problems will be considered, each is typical of the type of local problems which must be solved by the component controllers. Let D belong to B ; b and Y be in G_H ; and c, \tilde{c} be in X .

Local Optimization Problem I. (LOP I)

Minimize on $G_U \times G_H$: $J(u, h) = \frac{1}{2} N_H^2(A\beta u) + \frac{1}{2} N_H^2(C(y-Y))$

Subject to: $h = A(f+g+\beta u)$

$h(t) = b(t) \quad T-r \leq t \leq T$

$[Dh](T-r) = c$

$y = \gamma h$

given that $[(I-\gamma^*C^2\gamma)^{-1}Ag](T-r) = \tilde{c}$ and $D(I-\gamma^*C^2\gamma)^{-1}Ag = 0$.

Here the control objective is to force the response h to agree with b over the terminal interval while at the same time steering the response y along a desired trajectory Y . The conditions $[(I-\gamma^*C^2\gamma)^{-1}Ag](T-r) = \tilde{c}$ and $D(I-\gamma^*C^2\gamma)^{-1}Ag = 0$ summarize the information about g that allows the controller to solve the optimization problem. The rationale for these restrictions will become apparent in the proof. In the performance index for LOP I the term $\frac{1}{2} N_H^2(C(y-Y))$ corresponds to the standard quadratic cost functional $\frac{1}{2} \int_0^T (y-Y)^2 dt$ whereas the term $\frac{1}{2} N_H^2(A\beta u)$ is novel and represents a penalty on the response of the system rather than a direct penalty on the control energy.

In order to guarantee the existence of a feasible solution, we make the following assumption.

Regularity Assumption: Given c in X there exists a u in G_U and h in G_H such that $h = A\beta u$, $h(t) = 0$ for $T-r \leq t \leq T$, and $[Dh](T-r) = c$.

Recall that reproducing kernel Hilbert spaces are characterized by the fact that function evaluation is a continuous linear functional. Consequently, the constraint set for this problem is a closed convex set. A slight modification of the basic argument for minimum norm problems [5] yields the existence of a unique solution to LOP 1. In order to characterize this solution, LOP 1 can be reformulated as a optimization problem in the Hilbert space $G_U \times G_H$. It should be noted that the condition $h(t) = b(t)$ for $T-r \leq t \leq T$ is equivalent to the conditions $h(T) = b(T)$ and $(I - P_{T-r})(h - b) = 0$.

The Lagrange Multiplier Theorem asserts that if (u, h) minimizes J subject to the constraints then there exists elements λ and $\tilde{\kappa}$ of G_H and μ, v in X such that $Q_H(A\beta\delta u, A\beta u - \lambda) + Q_H(\delta h, \tilde{x}^* C^* C(\tilde{x}h - Y) + \lambda + (I - P_{T-r})\tilde{\kappa} + K(\cdot, T)\mu + D^*K(\cdot, T-r)v) = 0$ for all $(\delta u, \delta h)$ in $G_U \times G_H$. Let $\kappa = (I - P_{T-r})\tilde{\kappa}$ then $P_{T-r}\kappa = 0$. Using the controllability assumption, it follows that $A\beta u = (I - P_0)\lambda$ and $\tilde{x}^* C^* C(\tilde{x}h - Y) + \lambda + \kappa + K(\cdot, T)\mu + D^*K(\cdot, T-r)v = 0$. Since $[C^*h](t) = K(t, T)h(T) - K(0, T)h(0) - [Ch](t)$, one obtains after eliminating μ from the previous equation

$$\lambda = \tilde{x}^* C^2(\tilde{x}h - Y) + (1/(1+r))K(\cdot, T)\lambda(0) + (1/(1+r))K(\cdot, T)[D^*K(\cdot, T-r)v](0) - D^*K(\cdot, T-r)v - \kappa$$

and

$$(I - P_0)\lambda = \tilde{x}^* C^2(\tilde{x}h - Y) + k_1(\cdot, \lambda(0)) + k_2(\cdot, v) - \kappa \quad (3.1)$$

where $k_1(t, \lambda(0)) = [r_1(I-P_0)K(\cdot, T)\lambda(0)](t)$, $k_2(t, v) = [(I-P_0)\{r_1 k(\cdot, T)[D^*K(\cdot, T-r)v](0) - D^*K(\cdot, T-r)v\}](t)$ and $r_1 = 1/(1+r)$. Note that for fixed t in S $k_1(t, \lambda(0))$ and $k_2(t, v)$ are linear in $\lambda(0)$ and v . Since $h = A(f+g+\beta u) = A(f+g) + (I-P_0)\lambda$, it follows that

$$h = (I-\tilde{\tau}^*C^2\tilde{\tau})^{-1}\{A(f+g) - \tilde{\tau}^*C^2Y + k_1(\cdot, \lambda(0)) + k_2(\cdot, v) - \kappa\}. \quad (3.2)$$

The constraints imply $h = (I-P_{T-r})b + P_{T-r}h$. Also the causal properties of $(I-\tilde{\tau}^*C^2\tilde{\tau})^{-1}$ and $D(I-\tilde{\tau}^*C^2\tilde{\tau})^{-1}$ together with the fact that $P_{T-r}\kappa = 0$ imply that $P_{T-r}[(I-\tilde{\tau}^*C^2\tilde{\tau})^{-1}\kappa] = 0$ and $P_{T-r}[D(I-\tilde{\tau}^*C^2\tilde{\tau})^{-1}\kappa] = 0$. Using these observations we obtain the following linear system of equations for $\lambda(0)$ and v :

$$\begin{aligned} b(T-r) = & [(I-\tilde{\tau}^*C^2\tilde{\tau})^{-1}Af](T-r) + \tilde{c} - [(I-\tilde{\tau}^*C^2\tilde{\tau})^{-1}\tilde{\tau}^*C^2Y](T-r) \\ & + [(I-\tilde{\tau}^*C^2\tilde{\tau})^{-1}\{k_1(\cdot, \lambda(0)) + k_2(\cdot, v)\}](T-r) \end{aligned} \quad (3.3)$$

$$\begin{aligned} c = & [D(I-\tilde{\tau}^*C^2\tilde{\tau})^{-1}Af](T-r) - [D(I-\tilde{\tau}^*C^2\tilde{\tau})^{-1}\tilde{\tau}^*C^2Y](T-r) \\ & + [D(I-\tilde{\tau}^*C^2\tilde{\tau})^{-1}\{k_1(\cdot, \lambda(0)) + k_2(\cdot, v)\}](T-r). \end{aligned}$$

It is apparent that the conditions $[(I-\tilde{\tau}^*C^2\tilde{\tau})^{-1}Ag](T-r) = \tilde{c}$ and $D(I-\tilde{\tau}^*C^2\tilde{\tau})^{-1}Ag = 0$ allow us to solve for $\lambda(0)$ and v with limited knowledge of g . Using the constraint $(I-P_{T-r})(h-b) = 0$ and the fact that $\kappa = A(f+g) + k_1(\cdot, \lambda(0)) + k_2(\cdot, v) - (I-\tilde{\tau}^*C^2\tilde{\tau})h + \tilde{\tau}^*C^2Y$, we obtain from (3.1), (3.2) the following theorem.

THEOREM 2.3. The pair (u, h) in $G_U \times G_H$ solves LOP I if and only if the trajectory h has the form

$$h = (I-P_{T-r})b + P_{T-r}(I-\tilde{\tau}^*C^2\tilde{\tau})^{-1}\{A(f+g) - \tilde{\tau}^*C^2Y + k_1(\cdot, \lambda(0)) + k_2(\cdot, v)\} \quad (3.4)$$

and the control u has the form

$$u = w_1 + w_2 + w_3 \quad (3.5)$$

where

$$w_1 = \beta^{-1} A^{-1} \bar{x}^* C^2 (\bar{x} h - Y)$$

$$w_2 = \beta^{-1} A^{-1} (I - \bar{x}^* C^2 \bar{x}) (I - P_{T-r}) b - \beta^{-1} A^{-1} (I - \bar{x}^* C^2 \bar{x}) (I - P_{T-r}) (I - \bar{x}^* C^2 \bar{x})^{-1} \{A(f+g) - \bar{x}^* C^2 Y\}$$

$$w_3 = \beta^{-1} A^{-1} (I - \bar{x}^* C^2 \bar{x}) (P_{T-r}) (I - \bar{x}^* C^2 \bar{x})^{-1} \{k_1(\cdot, \lambda(0)) + k_2(\cdot, v)\}.$$

Here $\lambda(0)$, v satisfy the linear system of equations (3.3) and β^{-1} is the pseudoinverse of β .

In this result the control u is the sum of three terms, a dynamic feedback w_1 , a causal (non-anticipatory) function w_2 of the inputs b , f , g , Y , and a open loop term w_3 which depends on f , Y and the constants c and c , i.e., on some knowledge of g on the whole interval $[-r, T-r]$. For the implementation of such a control law it is assumed that the observations $\bar{x}h$ are available for feedback. The system diagram with such a control strategy implemented is as follows.

Figure 2

One should note for finite dimensional state-space systems that $S = [0, T]$, and $w_2 = 0$. A simple example illustrating LOP 1 is presented after we take up an alternate form of the component problem.

Next we consider a local optimization problem with the standard quadratic cost functional $\frac{1}{2} \int_0^T |u'|^2 dt + \frac{1}{2} \int_0^T |y - Y|^2 dt$.

Local Optimization Problem II. (LOP II)

Minimize on $G_U \times G_H$: $J(u, h) = \frac{1}{2} N_H^2(u) + \frac{1}{2} N_H^2(C(y-Y))$

Subject to: $h = f + Bh + g + \beta u$

$h(T) = b(T)$

$[Dh](T) = c$

$y = \gamma h$

given that $[\hat{A}g](T) = \tilde{c}$ and $D\hat{A}g = 0$ where $\hat{A} = (I - B - \beta\beta^*(I - \tilde{B})^{-1}\gamma^*C^2\gamma)^{-1}$ and $\tilde{B}h = B^*h - K(\cdot, T)[B^*h](0)$.

In this problem we must restrict attention to finite dimensional, state space systems. Let $B = \alpha C$ with α a linear transformation on X . With this restriction \tilde{B} as defined above is in B , hence $(I - \tilde{B})^{-1}$ exists, and we may again use the Lagrange Multiplier Theorem to characterize the optimal trajectory and control.

In order that (u, h) in $G_U \times G_H$ solve this problem it is necessary that there exist multipliers λ in G_H and μ, v in X such that $Q_H(\delta u, u) + Q_H(C\gamma\delta h, C(\gamma h - Y)) + Q_H(\delta h - B\delta h - \beta\delta u, \lambda) + \langle [\delta h](T), \mu \rangle + \langle [D\delta h](T), v \rangle = Q_H(u - \beta^*\lambda, \delta u) + Q_H(\gamma^*C^*C(\gamma h - Y) + (I - B)^*\lambda + K(\cdot, T)\mu + D^*K(\cdot, T)v, \delta h) = 0$ for all $(\delta u, \delta h)$ in $G_U \times G_H$. Consequently, $u = (I - P_0)\beta^*\lambda$ and $\gamma^*C^*C(\gamma h - Y) + (I - B)^*\lambda + K(\cdot, T)\mu + D^*K(\cdot, T)v = 0$. One obtains, eliminating μ from the above equation $\lambda = \gamma^*C^2(\gamma h - Y) + B^*\lambda + K(\cdot, T)\lambda(0) - K(\cdot, T)[B^*\lambda](0) + K(\cdot, T)[D^*K(\cdot, T)v](0) - D^*K(\cdot, T)v$. Thus $\lambda = (I - \tilde{B})^{-1}\{\gamma^*C^2(\gamma h - Y) + K(\cdot, T)\lambda(0) + K(\cdot, T)[D^*K(\cdot, T)v](0) - D^*K(\cdot, T)v\}$ and

$$(I - P_0)\lambda = (I - \tilde{B})^{-1}\gamma^*C^2(\gamma h - Y) + (I - P_0)(I - \tilde{B})^{-1}\{K(\cdot, T)\lambda(0) + K(\cdot, T)[D^*K(\cdot, T)v](0) - D^*K(\cdot, T)v\}. \quad (3.6)$$

It follows that

$$\begin{aligned}
h &= \hat{A}f + \hat{A}g - \hat{A}\beta\beta^*(I - \tilde{B})^{-1}\tilde{x}^*C^2Y + \hat{A}\beta\beta^*(I - P_0)(I - \tilde{B})^{-1}\{K(\cdot, T)\lambda(0) \\
&\quad + K(\cdot, T)[D^*K(\cdot, T)v](0) - D^*K(\cdot, T)v\} \\
&= \hat{A}f + \hat{A}g - \hat{A}\beta\beta^*(I - \tilde{B})^{-1}\tilde{x}^*C^2Y + k_{11}(\cdot, \lambda(0)) + k_{12}(\cdot, v)
\end{aligned} \tag{3.7}$$

and

$$u = \beta^*(I - \tilde{B})^{-1}\tilde{x}^*C^2(\tilde{x}h - Y) + k_{13}(\cdot, \lambda(0)) + k_{14}(\cdot, v) \tag{3.8}$$

where

$$\begin{aligned}
k_{13}(t, \lambda) &= [\beta^*(I - P_0)(I - \tilde{B})^{-1}K(\cdot, T)\lambda](t) \\
k_{14}(t, v) &= [\beta^*(I - P_0)(I - \tilde{B})^{-1}\{K(\cdot, T)[D^*K(\cdot, T)v](0) - D^*K(\cdot, T)v\}](t) \\
k_{11}(t, \lambda) &= [\hat{A}\beta k_{13}(\cdot, \lambda)](t) \\
k_{12}(t, v) &= [\hat{A}\beta k_{14}(\cdot, v)](t).
\end{aligned}$$

For fixed t , $k_{11}(t, \lambda)$, $k_{13}(t, \lambda)$ are linear in λ and $k_{12}(t, v)$, $k_{14}(t, v)$ are linear in v . The remaining constraints yield the following linear system of equations for $\lambda(0)$ and v :

$$b(T) = [\hat{A}f](T) + \tilde{c} - [\hat{A}\beta\beta^*(I - \tilde{B})^{-1}\tilde{x}^*C^2Y](T) + k_{11}(T, \lambda(0)) + k_{12}(T, v) \tag{3.9}$$

$$c = [D\hat{A}f](T) - [D\hat{A}\beta\beta^*(I - \tilde{B})^{-1}\tilde{x}^*C^2Y](T) + [D\{k_{11}(\cdot, \lambda(0)) + k_{12}(\cdot, v)\}](T).$$

Again the rationale for the conditions $[Ag](T) = c$ and $DAg = 0$ becomes apparent.

THEOREM 2.4. The pair (u, h) in $G_U \times G_H$ solves LOP II if and only if u and h are as given in equations (3.7), (3.8) and $\lambda(0)$, v satisfy equations (3.9).

Here the optimal control is made up of two terms: a dynamic feedback of the difference between the system observation and the desired trajectory and an open loop term depending upon the input f and the coordination

parameters c, \tilde{c} . The structure of the solution is depicted in the following diagram.

Figure 3

To illustrate the significance of LOP I and LOP II, we consider two examples.

Example 3.1 The problem is to steer the solution of the scalar equation $h'(t) = h(t-1) + g_1(t) + v(t)$, $t \geq 0$ with initial position $h(t) = 1$ for $-1 \leq t \leq 0$ to the terminal position $h(t) = 0$ for $1 \leq t \leq 2$. We assume limited information about g_1 . The system may be written as $h = f + g + Bh + u$ or $h = A(f+g+u)$ with $f(t) = 1$ for $-1 \leq t \leq 2$, $g = Cg_1$ and $u = Cv$. Here $A = (I-B)^{-1}$ with $[Bh](t) = 0$ for $-1 \leq t \leq 0$ and $[Bh](t) = \int_0^t h(\tau-1) d\tau$ for $0 \leq t \leq 2$. We assume $[(I-C^2)^{-1}Ag](1) = \tilde{c}$. A solution is obtained by setting $D = 0$, $b = 0$, $y = h$ and choosing an appropriate Y in LOP I. We obtain

$$[Af](t) = \begin{cases} f(t) & -1 \leq t \leq 0 \\ f(t) + \int_0^t f(\tau-1) d\tau & 1 \leq t \leq 2 \\ f(t) + \int_0^t f(\tau-1) d\tau + \int_1^t (t-\tau) f(\tau-2) d\tau & 0 \leq t \leq 2 \end{cases}$$

$$[A^{-1}f](t) = f(t) - [Bf](t) \quad -1 \leq t \leq 2$$

$$[C^2h](t) = \begin{cases} 0 & -1 \leq t \leq 0 \\ \int_0^t (t-\tau) h(\tau) d\tau & 0 \leq t \leq 2 \end{cases}$$

$$[(I-C^2)^{-1}f](t) = \begin{cases} f(t) & -1 \leq t \leq 0 \\ f(t) + \int_0^t \sinh(t-\tau) f(\tau) d\tau & 0 \leq t \leq 2 \end{cases}$$

and the optimal trajectory and control can be determined from equations

(3.3), (3.4), and (3.5).

Example 3.2 Let $B = C$ and consider the problem of steering the scalar system $\dot{h}(t) = t + [Bh](t) + 2u(t)$ for $0 \leq t \leq 2$ from $h(0) = 0$ to $h(2) = 1$ subject to the constraint $[Dh](T) = (1/2) \int_0^2 h(t) dt = 0$. Setting $g = 0$ and $y = \frac{1}{2}h$ and choosing an appropriate Y , LOP II may be used to solve this problem. One obtains, using Laplace transforms,

$$[Bh](t) = th(0) - \int_0^t h(\tau) d\tau$$

$$[(I-\tilde{B})^{-1}h](t) = h(t) - \int_0^t e^{-(t-\tau)} h(\tau) d\tau + (1-e^{-t})h(0)$$

$$[\hat{A}h](t) = h(t) + \int_0^t [\sqrt{2} \sinh \sqrt{2}(t-\tau) + \cosh \sqrt{2}(t-\tau)] h(\tau) d\tau$$

$$[D^*K(\cdot, T)v](t) = [DK(\cdot, t)v](T) = (1+t-t^2/4)v$$

and the optimal trajectory and control are determined from equations (3.7), (3.8), and (3.9).

These examples are indicative of the use of LOP I and LOP II. Frequently Laplace transforms may be used to determine system operators. In more general problems, our Hilbert space setting aids in the representation and approximation of system operators [2], [12].

4. A Decentralized Control Strategy. We will present a decentralized control strategy, a scheme for coordinating local control decisions, for large scale systems in which component interactions are suitably limited. First, consider the two component control system

$$\dot{h}_1 = f_1 + B_1 h_1 + C_2 h_2 + \beta u_1$$

$$\dot{h}_2 = f_2 + C_1 h_1 + B_2 h_2 + \beta u_2$$

where the B 's and C 's are from B . We can diagram the system as follows:

Figure 4

where $A_i = (I - B_i)^{-1}$ for $i = 1, 2$.

We define two components to be stably connected provided for all g in $P_0 G_H$ the condition $C_i(I - C^2)^{-1} A_i C_j g = 0$ holds for $\{i, j\} = \{1, 2\}$. Large scale systems composed of several components, each pair of which is stably connected, will be referred to as stably connected systems. Local control laws for stably connected systems can be coordinated by imposing side conditions as in the previous section and exchanging that information among the component controllers.

For a stably connected two component system, consider the problem of local controllers choosing controls u_1 and u_2 , respectively, on the interval $0 \leq t \leq T$ which steer h_1 and h_2 to zero over the interval $T - r \leq t \leq T$, i.e. LOP I with $b = Y = 0$ and $\beta = \alpha = 1$. We are led to the following control coordination strategy.

Coordination Strategy: Let $\{i, j\} = \{1, 2\}$. Given elements c_1 and c_2 from X and f_1 and f_2 in G_H the i th controller solves LOP I with $A = A_i$, $g = C_j h_j$, $D = D_i = (I - C^2)^{-1} A_j C_i$, $c = c_i$, and $\tilde{c} = c_j$.

In this strategy the condition $[(I - C^2)^{-1} A_1 C_2 h_2](T - r) = c_2$ represents the information that the first controller must have about the second component in order that the local optimization problem can be solved. This condition becomes a constraint on the second component. Similarly, the condition $[(I - C^2)^{-1} A_2 C_1 h_1](T - r) = c_1$ becomes a constraint on the first component. The stably connected assumption implies that $D_i(I - C^2)^{-1} A_i g = 0$ for each component and, hence, each controller can solve its local optimization problem.

The stably connected condition can be given a geometric interpretation in specific cases. If $C_i = 0$ for $i = 1$ or 2 then the

components are stably connected and there is a hierarchical relation between the components. If C_i commutes with $(I-C^2)^{-1}$ for $i = 1$ and 2 on $P_0 G_H$ then the stably connected condition is equivalent to the condition $C_i A_i C_j = 0$ for $\{i,j\} = \{1,2\}$. If the latter condition is not satisfied, one could argue that the input-output operator for the first component should be $(I-A_1 C_2 A_1 C_1)^{-1} A_1$ rather than A_1 . Similarly for the second component. For the examples considered in this paper $C_i = \alpha C$ with $r > 0$ or $r = 0$, where α is a $d \times d$ matrix, in either case C_i commutes with $(I-C^2)^{-1}$ on $P_0 G_H$. For finite dimensional state space systems i.e., $r = 0$ and $B_1 = \alpha_{11} C$, $C_2 = \alpha_{12} C$, $C_1 = \alpha_{21} C$, and $B_2 = \alpha_{22} C$ with the α_{ij} matrices of appropriate dimensions the two components will be stably connected if and only if the controllability subspace determined by $\{\alpha_{ii}, \alpha_{ij}\}$ is a subspace of the null space of α_{ji} for $\{i,j\} = \{1,2\}$.

Other combinations of LOP I or LOP II can be used as the basis for defining a coordination strategy. In any such strategy, if D_i denotes the operator appearing in the additional component constraint then we must extend the definition of stably connected components to mean $D_1 D_2 = D_2 D_1 = 0$. Frequently the components form a hierarchy in which $D_1 = 0$ or $D_2 = 0$ and this condition is automatically satisfied.

Example 4.1 Consider the decentralized control problem of steering the two component system

$$h_1'(t) = h_1(t-1) + h_2(t) + v_1$$

$$h_2'(t) = h_2(t) + v_2$$

from $h_1(t) = 1$ for $-1 \leq t \leq 0$ to $h_1(t) = 0$ for $1 \leq t \leq 2$ and $h_2(0) = 0$ to $h_2(2) = 1$. With notation as in Examples 3.1 and 3.2 the system may be written as

$$h_1 = f + B_1 h_1 + C h_2 + u_1$$

$$h_2 = B_2 h_2 + u_2.$$

Clearly the components form a hierarchy. The following coordination strategy is suggested: the controller for the first component uses LOP I, as in Example 3.1 with $g = Ch_2$, to determine u_1 while the second controller uses LOP II, as in Example 3.2 with $D = (I - C^2)^{-1}A_1$, to determine u_2 .

The implementation of our coordination strategies for a two component system requires that appropriate values for the parameters c_1 and c_2 be available. These parameters are referred to as coordination parameters and the problem of determining values for these parameters leads to a higher level coordination problem.

A coordinating strategy can be developed for any stably connected system. A rich class of such systems are those which can be thought of as hierarchies. A canonical example is a multi-component system with signal diagram of the following form

Figure 5

Clearly each pair of components is stably connected and we can envision a command structure where the controller for the first component sets the coordination parameters and, hence, the side conditions for the other two components, i.e., specifies the values of $[(I - C^2)^{-1}A_1C_2h_2](T)$ and $[(I - C^2)^{-1}A_1C_3h_3](T)$ if LOP I is used. More generally, we can consider a grid network of the form

Figure 6

provided the system is stably connected. The side conditions, using LOP

I , would have the following form.

$$\text{On the first component: } [(I-C^2)^{-1}A_2C_1h_1](T) = c_1$$

$$\text{On the second component: } [(I-C^2)^{-1}A_1C_{21}h_2](T) = c_{21}$$

$$\text{and } [(I-C^2)^{-1}A_3C_{23}h_2](T) = c_{23}$$

$$\text{On the third component: } [(I-C^2)^{-1}A_2C_3h_3](T) = c_3.$$

Physical limitations on component interactions can yield stably connected systems or decomposition methods may be used to transform a model into a stably connected system. We now present a basic decomposition procedure for finite dimensional systems of the form $h(t) = f(t) + \alpha_0^t h(s)ds + \beta_1 u_1 + \beta_2 u_2$ where $r = 0$ and α is a $d \times d$ matrix and β_i is a $d \times n_i$ matrix for $i = 1, 2$. See [11] for examples of this decomposition and related coordination problems. For $i=1, 2$ let Π_i denote the projection of X onto the controllability subspace spanned by the columns of the matrix $\{\beta_i, \alpha\beta_i, \dots, \alpha^{d-1}\beta_i\}$ and Π_3 a projection of X onto $\Pi_1 X \cap \Pi_2 X$. Assume $X = \Pi_1 X + \Pi_2 X$ and recall that the identity may be written as $I = \Pi_1 + \Pi_2 - \Pi_3$.

We want a decomposition of the system in terms of the Π 's to which we can apply our distributed control strategy. For $0 \leq m_0, m_1, m_2 \leq 1$, let

$$f_1 = (\Pi_1 - m_0 \Pi_3)f \quad f_2 = (\Pi_2 - (1-m_0)\Pi_3)f$$

and

$$\begin{aligned} \alpha_{11} &= (\Pi_1 - m_1 \Pi_3)\alpha\Pi_1 & \alpha_{12} &= m_2 \Pi_3 \alpha \Pi_2 \\ \alpha_{21} &= m_1 \Pi_3 \alpha \Pi_1 & \alpha_{22} &= (\Pi_2 - m_2 \Pi_3)\alpha \Pi_2. \end{aligned}$$

Furthermore, let $B_1 = \alpha_{11}C$, $C_1 = \alpha_{21}C$, $B_2 = \alpha_{22}$, $C_2 = \alpha_{12}C$. Our decomposition has the form

$$\begin{aligned} h_1 &= f_1 + B_1 h_1 + C_2 h_2 + \beta_1 u_1 \\ h_2 &= f_2 + C_1 h_1 + B_2 h_2 + \beta_2 u_2. \end{aligned}$$

It follows, using the invariance properties of the controllability subspaces,

that

$$\begin{aligned}
 h_1 + h_2 &= f_1 + f_2 + (\alpha_{11} + \alpha_{21}) \int_0^t h_1 dt + (\alpha_{12} + \alpha_{22}) \int_0^t h_2 dt + \beta_1 u_1 + \beta_2 u_2 \\
 &= f + \Pi_1 \alpha \Pi_1 \int_0^t h_1 dt + \Pi_2 \alpha \Pi_2 \int_0^t h_2 dt + \beta_1 u_1 + \beta_2 u_2 \\
 &= f + \Pi_1 \alpha \int_0^t h_1 dt + \Pi_2 \alpha \int_0^t h_2 dt + \beta_1 u_1 + \beta_2 u_2 \\
 &= f + (\Pi_1 - \Pi_3) \alpha \int_0^t h_1 dt + \Pi_3 \alpha \int_0^t h_1 dt \\
 &\quad + (\Pi_2 - \Pi_3) \alpha \int_0^t h_2 dt + \Pi_3 \alpha \int_0^t h_2 dt + \beta_1 u_1 + \beta_2 u_2 \\
 &= f + (\Pi_1 + \Pi_2 - \Pi_3) \alpha \int_0^t (h_1 + h_2) dt + \beta_1 u_1 + \beta_2 u_2.
 \end{aligned}$$

Thus $h = h_1 + h_2$ and the decomposition can be diagrammed as follows:

Figure 7

where $A_i = (I - B_i)^{-1}$, $i=1,2$.

The system will be stably connected provided $C_1 A_1 C_2 = C_2 A_2 C_1 = 0$. Here $C_2 A_2 C_1 = m_1 \Pi_3 \alpha \Pi_1 (I - (\Pi_2 - m_2 \Pi_3) \alpha \Pi_2 C)^{-1} m_2 \Pi_3 \alpha \Pi_2 C$ and a similar expression holds for $C_1 A_1 C_2$.

We consider three cases i) $\Pi_1 X \cap \Pi_2 X = \{0\}$; ii) $\Pi_2 X \subseteq \Pi_1 X$; and iii) $\Pi_1 X \neq X$, $\Pi_2 X \neq X$, $\Pi_1 X \cap \Pi_2 X \neq \{0\}$. If $y = (I - (\Pi_2 - m_2 \Pi_3) \alpha \Pi_2 C)^{-1} m_2 \Pi_3 \alpha \Pi_2 x$ then the invariance properties of the controllability subspaces imply that $\Pi_1 C y = \Pi_3 C y$. Consequently, the stably connected condition is satisfied for all choices of m_1 and m_2 whenever $\Pi_3 \alpha \Pi_3 = 0$. In the first case $\Pi_3 = 0$ and there is no interaction between the components. In the second case $\Pi_3 = \Pi_2$ and the situation seems to be most interesting when $\Pi_3 \alpha \Pi_3 \neq 0$. In this case the stably connected condition can be met by choosing either m_1 or m_2 to be zero forming a "hierarchical" system. Finally, the third case is mixed since $\Pi_3 \alpha \Pi_3$ might or might not be zero. When $\Pi_3 \alpha \Pi_3 = 0$ and $m_1 m_2 \neq 0$ note that the system is not a hierarchy and the components interact.

5. Concluding Remarks. If one views the system components as basic

building blocks then our coordination strategy for stably connected systems may be viewed as a rule for putting together more complex structured systems. From this point of view the open and closed loop control laws arising from the local optimization problems determine additional components to be "hardwired" into the system.

To implement such a control strategy one may envision a coordinator who acts or may act on the system by setting the component objective: trajectories, or side conditions necessary for coordination. terminal constraints, desired trajectories, or side condition necessary for coordination. Such a coordinator may use centralized knowledge of the performance of each component to set coordination parameters. Realistically, only limited information on component performance would be available to the coordinator. Another possibility is that a command structure is imposed among the the system components for the choice of component objectives and the passage of information. There are many possibilities. However, the basic idea is that in any such structure there is a predetermined order in which component objectives are to be set. Certain components may be free to set their own objectives while other components would have extra side conditions added to their requirements.

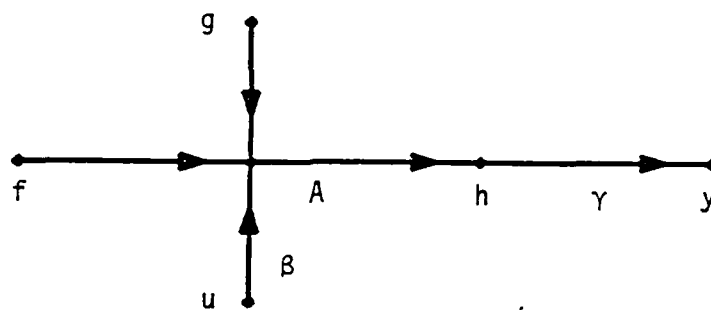


Figure 1

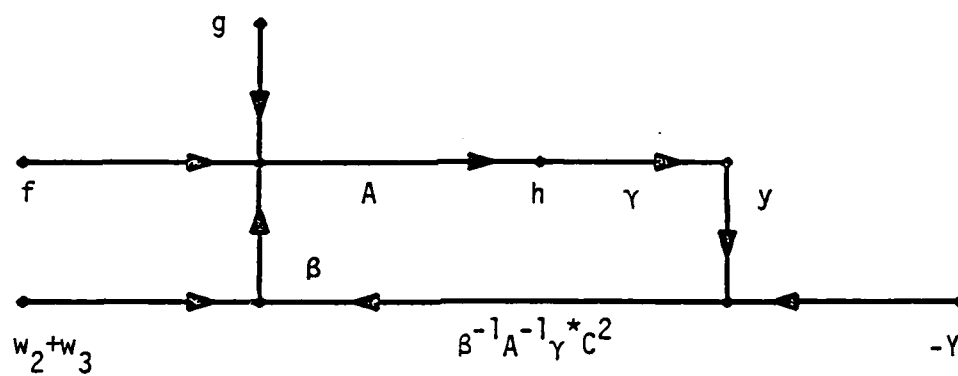


Figure 2

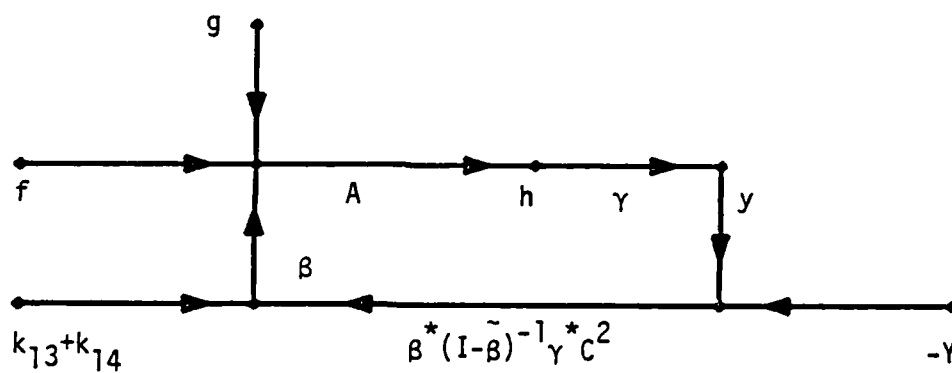


Figure 3

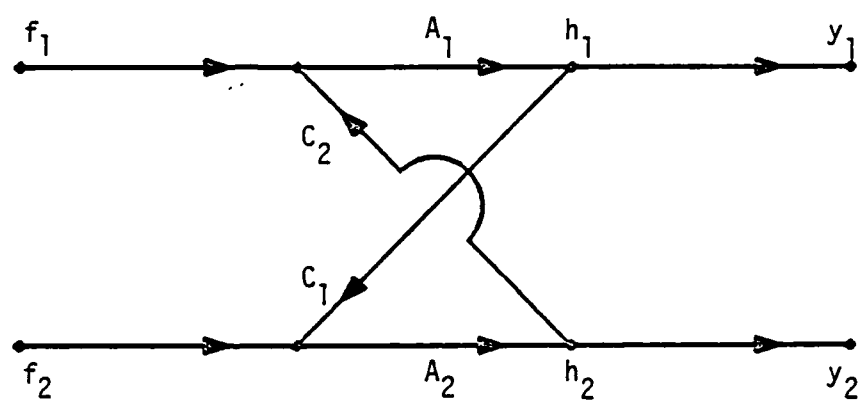


Figure 4

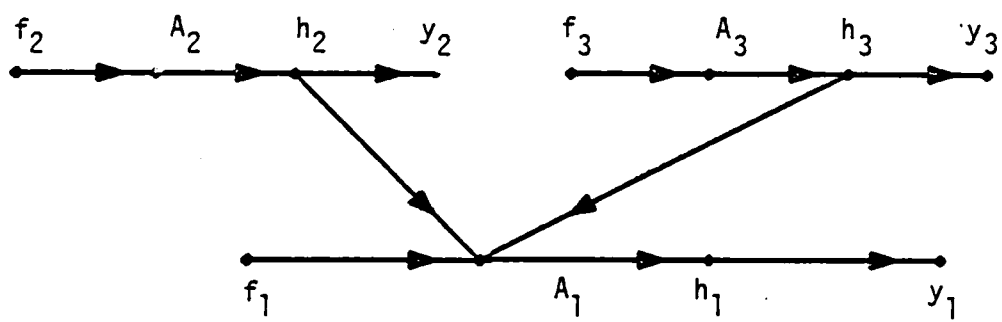


Figure 5

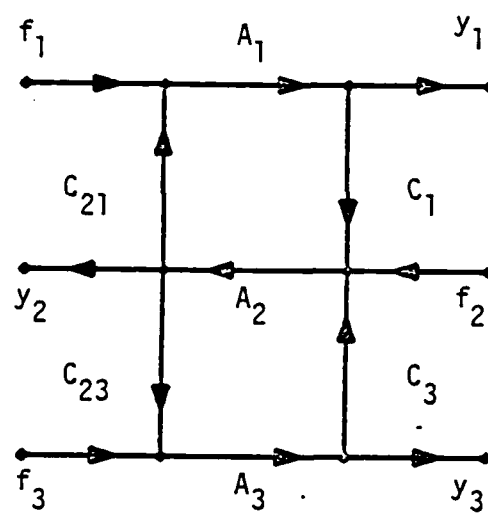


Figure 6

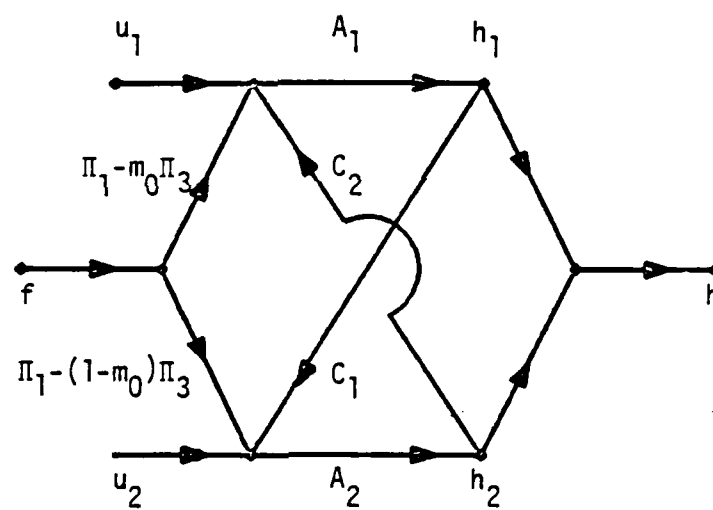


Figure 7

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CONTROL COORDINATION FOR LARGE SCALE SYSTEMS

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ABSTRACT

A control strategy, which allows some level of autonomous component control, for large scale systems is presented. The approach requires that component interactions be suitably limited so that variational methods can be used to determine component controls.

INTRODUCTION

Decentralized control problems for large scale finite dimensional linear systems are considered. For systems with two controllers we consider the problem of independently choosing controls to steer component responses to desired terminal conditions. It is shown that some level of autonomous component control is possible when additional constraints are placed upon the component requirements and exchanges of related information between components is allowed. The ability to coordinate locally designed control laws depends on the level of interaction between system components.

Our methods may be referred to as multi-level or hierarchical, (1), (2), in the sense that 1) the system must be decomposed into local components, ii) local models are used in the design of component control laws, iii) a higher level coordination problem is introduced. Typically multi-level methods involve off line iterative calculations and result in open loop controls. In our approach coordination of component controls is achieved in one step by the imposition of additional constraints on the system components and closed loop control laws are obtained. The choice of appropriate values for parameters appearing in these added constraints leads to a higher level coordination problem.

COMPONENT CONTROL PROBLEM

Our methods require that components modelled by differential equations be written in integrated form as $h(t) = f(t) + [Bh](t) + \int_0^t u(\tau) d\tau + g_1(t)$ and $y(t) = \gamma_1 h(t) + \gamma_2 g_2(t)$, where $[Bh](t) = \alpha[Ch](t)$

with $[Ch](t) = \int_0^t h(\tau) d\tau$ and $\alpha, \beta, \gamma_1, \gamma_2$ denote matrices of appropriate dimension. Here u denotes the component control, f denotes a known disturbance and g_1, g_2 denote the effect of other components upon the given component. Let $A = (I-B)^{-1}$ denote the component input-output operator.

We consider the problem of steering $h(t)$ from $h(0) = f(0)$ to a desired terminal condition $h(T) = b$. Let c, \tilde{c} denote fixed parameters and let D denote a linear operator defined on the underlying function space. The typical form of a component optimization problem is as follows:

$$\begin{aligned} \text{Minimize: } J(u, h) &= \frac{1}{2} \int_0^T (y - Y)^2 + (u')^2 d\tau \\ \text{Subject to: } h(t) &= f(t) + [Bh](t) + \int_0^t u(\tau) d\tau + g_1(t) \\ h(T) &= b \\ [Dh](T) &= c \\ y(t) &= \gamma_1 h(t) + \gamma_2 g_2(t) \end{aligned}$$

$$\begin{aligned} \text{given } [\hat{A}(g_1 + \hat{B}\gamma_2 g_2)](T) &= \tilde{c} \text{ and } D(\hat{A}(g_1 + \hat{B}\gamma_2 g_2)) = 0 \\ \text{where operators } \hat{A} \text{ and } \hat{B} \text{ are defined by} \\ \hat{A} &= (I - B - \hat{B}\gamma_1)^{-1}, \hat{B} = \beta \hat{B}^* (I - \tilde{B})^{-1} \gamma_1^* C^2, \text{ and} \\ [Bh](t) &= [B^*h](t) - K(t, T)[B^*h](0). \end{aligned}$$

In this problem an interval constraint has been placed upon h , Y denotes a desired response, and $K(s, t) = k(\min(s, t))I$ with I the identity and $k(t) = 1+t$. The conditions $[\hat{A}(g_1 + \hat{B}\gamma_2 g_2)](T) = \tilde{c}$ and $D(\hat{A}(g_1 + \hat{B}\gamma_2 g_2)) = 0$ summarize information about g_1 and g_2 that allow the controller to solve this optimization problem. These conditions will become added constraints on the components which determine g_1 and g_2 . For a similar reason the constraint $[Dh](T) = c$ is included in the above problem. For certain hierarchical relations between system components the condition $D(\hat{A}(g_1 + \hat{B}\gamma_2 g_2)) = 0$ is naturally satisfied.

The local control problem can be reformulated as a constrained optimization problem in a reproducing kernel Hilbert space of functions. The inner product in this space is essentially defined by

$$\langle f, g \rangle_H = \langle f(0), g(0) \rangle + \int_0^T \langle f'(t), g'(t) \rangle dt.$$

Lagrange multiplier methods (3), yield the optimal control

* Research supported in part by the Air Force Office of Scientific Research under Grant AFOSR-84-0236.

$$u(t) = [B^*(I-\tilde{B})^{-1}Y_1^*C^2(Y_1h+Y_2g_2-Y)](t) + k_1(t,\lambda) + k_2(t,v)$$

where the open loop terms are defined by

$$k_1(t,\lambda) = [B^*(I-P_0)(I-\tilde{B})^{-1}K(\cdot,T)\lambda](t)$$

$$k_2(t,v) = [B^*(I-P_0)(I-\tilde{B})^{-1}[K(\cdot,T)[D^*K(\cdot,T)v](0) - D^*K(\cdot,T)v]](t).$$

With this control strategy implemented the system response is

$$h(t) = \hat{A}f + \hat{A}(g_1 + \hat{B}Y_2g_2) - \hat{A}\hat{B}Y + \hat{A}B(k_1(\cdot,\lambda) + k_2(\cdot,v)).$$

and the multipliers λ, v are solutions of the equations

$$b = [\hat{A}f](T) + \tilde{c} - [\hat{A}\hat{B}Y](T) + [\hat{A}B(k_1(\cdot,\lambda) + k_2(\cdot,v))](T)$$

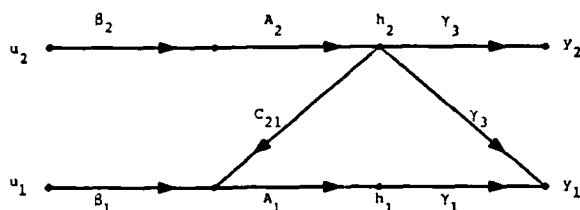
$$c = [D\hat{A}f](T) - [D\hat{A}\hat{B}Y](T) + [D\hat{A}B(k_1(\cdot,\lambda) + k_2(\cdot,v))](T).$$

Here $[(I-P_0)f](t) = f(t) - f(0)$ and $[B^*h](t) = \alpha^*[K(t,T)h(T) - K(t,0)h(0) - [Ch](t)]$. In the scalar case $[D^*K(\cdot,T)v](t) = [DK(\cdot,T)v](T)$ and in the vector case $[D^*K(\cdot,T)v](t)$ is determined by $\langle [D^*K(\cdot,T)v](t), e_j \rangle = \langle [DK(\cdot,T)v](T), v \rangle$.

For low order systems all operators arising in the component control problem can be obtained using Laplace transform methods. Higher order systems require numerical approximations to these operators. Theoretical verification of the existence and uniqueness of solutions to local control problems of the above form is the subject of a forthcoming paper (4).

CONTROL COORDINATION

As an example of our coordination strategy we consider the following two component system



Here $A_1 = (I-B_1)^{-1}$ with $B_1 = \alpha_1 C$ and $C_{21} = \alpha_{12} C$ where α_1 and α_{12} are matrices of appropriate dimension for $i = 1, 2$. The control problem is to choose u_1 and u_2 to steer h_1 and h_2 from given initial positions to desired terminal conditions $h_1(T)=b_1$ and $h_2(T)=b_2$ with limited information exchanged between components.

The following coordination strategy is suggested. The first controller solves the local control problem with $g_1=C_{21}h_2$, $g_2=Y_2h_2$, and $D=0$. This strategy forces the constraint $[\hat{A}_1(C_{21}h_2+B_1Y_2h_2)](T) = \tilde{c}$ on the second component. Also, the second controller can obtain a control to meet the desired objective by solving the local control problem with $g_1=g_2=0$ and the constraint

$$[Dh_2](T) = [\hat{A}_1(C_{21}h_2+B_1Y_2h_2)](T) = \tilde{c}.$$

For example, to steer h_1 from $h_1(0)=0$ to $h_1(T)=b_1$ and h_2 from $h_2(0)=0$ to $h_2(T)=b_2$, the component optimization problems would be

$$I. \text{ Minimize: } \frac{1}{2} \int_0^T (Y_1h_1 + Y_2h_2 - Y_1)^2 dt + \frac{1}{2} \int_0^T u_1^2 dt$$

$$\text{Subject to: } h_1 = B_1h_1 + C_{21}h_2 + B_1u_1$$

$$h_1(T) = b_1$$

$$y = Y_1h_1 + Y_2h_2$$

$$\text{given } [\hat{A}_1(C_{21}+\hat{B}_1Y_2)h_2](T) = \tilde{c},$$

$$II. \text{ Minimize: } \frac{1}{2} \int_0^T (Y_3h_2 - Y_2)^2 dt + \frac{1}{2} \int_0^T u_2^2 dt$$

$$\text{Subject to: } h_2 = B_2h_2 + B_2u_2$$

$$h_2(T) = b_2$$

$$[\hat{A}_1(C_{21}+\hat{B}_1Y_2)h_2](T) = \tilde{c}$$

$$y_2 = Y_3h_2.$$

Note that each local control problem can be solved independently provided that the operator $\hat{A}_1(C_{21}+\hat{B}_1Y_2)$ and the value of \tilde{c} is known to each controller. The choice of appropriate values for this coordination parameter leads to a higher level coordination problem.

This example illustrates our approach to decentralized control and control coordination for large scale systems. In the scalar case, operators which appear in this coordination strategy have the following representation

$$[\tilde{B}h](t) = \alpha(th(0) - [Ch](t))$$

$$[(I-B)^{-1}h](t) = (1-e^{-\alpha t})h(0) + h(t)$$

$$- \alpha \int_0^t e^{-\alpha(t-\tau)} h(\tau) d\tau$$

$$[C^2h](t) = \int_0^t (t-\tau)h(\tau) d\tau$$

$$[\hat{B}h](t) = (1/\alpha) \int_0^t (1-e^{-\alpha(t-\tau)})Yh(\tau) d\tau$$

and A is the input output operator with transfer function $(s^2+\alpha s)/(s^2-(\alpha^2+\beta^2Y^2))$. These formula can be used to simulate our coordination strategy for two component systems as diagrammed.

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RKH SPACE METHODS FOR APPROXIMATING THE COVARIANCE KERNELS
OF A CLASS OF STOCHASTIC LINEAR HEREDITARY SYSTEMS, II

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Reproducing kernel Hilbert space methods have produced useful approximations for estimation and control problems of deterministic linear hereditary systems. By obtaining explicit Parzen type RKH space representations of stochastic processes governed by linear hereditary systems as spaces of Hellinger integrable functions these same approximation methods apply to the problem of finding the covariance kernel given the model. Some sample calculations will be presented.

INTRODUCTION

A convenient summary of the inner product space geometry of second order stochastic processes can be found in [3, Chapter 2]. Our concern is with the covariance kernel of such a process, the reproducing kernel of a Hilbert space representation of the process [5]. A recent collection of papers [9] covering the last two decades illustrates the utility of RKH space methods for computing various functionals of a process related to problems of estimation and detection.

The covariance kernel also plays a central role in the design of controls for linear finite dimensional state space systems [8]. In this work the diagonal of the kernel function, the variance of the system response to a general noise process, is most important and can be found by solving a matrix Riccati equation. However, for hereditary systems, systems modelled by delay differential equations for instance, the covariance kernel has not been so useful. One difficulty is that infinite dimensional state space methods lead to an operator Riccati equation for the variance of the states [2] rather than the variance of the process.

The approach of this paper to the problem of estimating the covariance kernel of the response of an hereditary system to a general noise process input is in the spirit of Parzen's original idea. We extend the method presented in [7] for scalar systems to obtain an explicit description of the reproducing kernel Hilbert space representing the process as a space of Hellinger integrable functions [4]. Convergence of a class of finite dimensional approximations to the covariance kernel is obtained using the approximation theory developed in [6].

SOME BACKGROUND NOTATION

We are concerned with second order processes $\{X(t), t \text{ in } S\}$ with values in R^d , i.e., the components of the $d \times d$ matrix $E(X(t)X'(t))$ are finite for each t . Let $\langle \cdot, \cdot \rangle$ denote the usual inner product for R^d and $|\cdot|$ the inner product norm.

Parzen's covariance kernel R is defined by $R(s,t) = E(X(s)X'(t))$, for each s and t in S . One has immediately that $\sum_{p,q=0}^n \langle c_p, R(s_p, s_q) c_q \rangle \geq 0$ for each sequence $\{s_p\}_{p=0}^n$ in S and sequence $\{c_p\}_{p=0}^n$ in R^d . Aronszajn's classic result [1] tells us

that there is a complete Hilbert space G for which R is a reproducing kernel, i.e., denoting the inner product by $Q(\cdot, \cdot)$ we have

- i) $R(\cdot, t)c$ is in G for each t in S and c in R^d and
- ii) $Q(f, R(\cdot, t)c) = \langle f(t), c \rangle$ for each f in G , t in S , and c in R^d .

It is appropriate to think of $\{G, Q\}$ as a representation of $\{X(t), t \text{ in } S\}$ since, for each c in R^d , the function ψ_c from $L_2(X(t), t \text{ in } S)$ into $\{G, Q\}$ defined by $\psi_c(u) = E(Xu')c$ maps $X(t)$ to $R(\cdot, t)c$, for each t in S . However, obtaining $\{G, Q\}$ explicitly is basic to making use of this representation of the process.

Consider for instance the process given by the real valued stochastic differential equation $dX = \alpha X(t)dt + dW$ where W is the standard Wiener process. The covariance kernel for $\{X(t), t \geq 0\}$ is given by $R(s,t) = (1/\alpha) \exp(\alpha \max(t,s)) \cdot \sinh(\alpha \min(t,s))$, for s, t in $S = [0, \infty)$. The corresponding RKH space G is the space of all functions f which are 0 at 0, absolutely continuous on compact subintervals of S , and such that

$\int_0^\infty |f'(t) - \alpha f(t)|^2 dt < \infty$. The inner product Q is given by

$$Q(f,g) = \int_0^\infty (f'(t) - \alpha f(t))(g'(t) - \alpha g(t))dt \text{ for each } f \text{ and } g \text{ in } G.$$

This elementary result can be generalized to include delay systems. However, Parzen's ideas apply to more complicated systems which we will attempt to encompass by introducing an abstract formulation. Assume $S = [-r, \infty)$ where $r \geq 0$ and k is an increasing function from S to the numbers with $k(-r)=0$. Let G_0 denote the space of continuous R^d valued functions on S and G_H the subset of G_0 to which f belongs only in case $f(-r) = 0$ and f is Hellinger integrable with respect to k on compact subintervals of S [4,6], i.e., for each compact subinterval $[u,v]$ of S there is a number b such that $\sum_s |df|^2/dk = \sum_{p=1}^m |f(s_p) - f(s_{p-1})|^2 / (k(s_p) - k(s_{p-1})) \leq b$ for each partition

$\{s_p\}_{p=0}^m$ of $[u,v]$. The Hellinger integral $\int_u^v |df|^2/dk$ is the least

such number b .

Note that elements of G_H are absolutely continuous on compact subintervals of S . Furthermore, if each of f and g is in G_H and $[u,v]$ is a compact subinterval

then the integral $\int_u^v \langle df, dg \rangle / dk$ exists as a limit through refinement of partitions of $[u, v]$ of approximating sums of the form $\sum_s \langle df, dg \rangle / dk = \sum_{p=1}^m \langle f(s_p) - f(s_{p-1}), g(s_p) - g(s_{p-1}) \rangle / (k(s_p) - k(s_{p-1}))$.

For each u in S and f in G_0 let $N_u(f) = \text{l.u.b. } |f(x)|$ and $x \leq u$

$$[P_u f](x) = \begin{cases} f(x) & x \leq u \\ f(u) & u \leq x. \end{cases}$$

Let \mathcal{B} denote the class of linear transformations of G_0 to which B belongs only in case

- i) $[Bf](u) = 0$ for each f in G_0 and $u < 0$ and
- ii) for each compact subinterval S_0 of \bar{S} there is a number c such that $|[Bf](v) - [Bf](u)| \leq c \int_u^v N_x(f) dk(x)$, for each f in G_0 and subinterval $[u, v]$ of S_0 .

Note that the formal integral operator

$$[Bf](u) = \begin{cases} 0 & u \leq 0 \\ \alpha \int_0^u f(s) dk(s) + \beta \int_0^u f(s-r) dk(s) & 0 \leq u, \end{cases}$$

where α and β are $d \times d$ matrices, restricted to G_0 is in \mathcal{B} .

Let \mathcal{A} denote the space of linear transformations of G_0 to which A belongs only in case $A - I$ is in \mathcal{B} , where I also denotes the identity transformation on G_0 . If B is in \mathcal{B} then $I - B$ is a reversible function from G_0 onto G_0 and $(I - B)^{-1}$ is in \mathcal{A} . If A is in \mathcal{A} then A is a reversible function from G_0 onto G_0 and $I - A^{-1}$ is in \mathcal{B} .

Consider the process

$$X(t) = \begin{cases} Z(t) & t \leq 0 \\ Z(t) + [BX](t) & 0 \leq t \end{cases}$$

$$= [AZ](t),$$

where B is in \mathcal{B} , $A = (I - B)^{-1}$, and Z is a mean-zero process in R^d with independent increments, and $E(dZ(dZ)') = (dk)I$, I the $d \times d$ identity. We assume that $Z(-r) = 0$. In the next section we want to obtain explicitly an RKH space which represents the process $\{X(t), t \text{ in } S\}$ in the sense of Parzen.

AN RKH SPACE PARAMETERIZATION OF R.

We begin with a generalization of the earlier elementary result for the process given by $X(t) = W(t) + \alpha \int_0^t X(s)ds$. Let G_∞ denote the subset of G_H to which f belongs only in case $\int_{-r}^\infty |df|^2/dk < \infty$ and Q_∞ the inner product for G_∞ given by $Q_\infty(f,g) = \int_{-r}^\infty \langle df, dg \rangle / dk$. Recall that elements of G_H are 0 at $-r$.

Theorem. If $R(\cdot, t)c$ is in G_H for each t in S and c in R^d then the process $\{X(t), t \text{ in } S\}$ given above can be represented in the sense of Parzen by the RKH space $\{G, Q\}$ where

- 1) G is the subspace of G_H to which f belongs only in case $(I-B)f$ is in G_∞ and
- 2) Q is the inner product for G given by $Q(f,g) = Q_\infty((I-B)f, (I-B)g)$ for each (f,g) in $G \times G$.

Indication of Proof. As an indication that everything works, notice that there is a function M from $S \times S$ to the $d \times d$ matrices such that $X(t) = [AZ](t) = \int_{-r}^t M(t,s)dZ(s)$ for each t in S and

$$E[dZ(u,v)([AZ](t))'] = \begin{cases} 0 & \text{if } t \leq u \leq v \\ \int_u^v M'(t,s)dk(s) & \text{if } u \leq v \leq t. \end{cases}$$

Also if $s \leq t$ and c in R^d then

$$\begin{aligned} [(I-B)R(\cdot, s)c](t) &= [(I-B)E(X(\cdot) X'(s))c](t) \\ &= [E(Z(\cdot) X'(s))c](t) \\ &= [E(Z(\cdot) X'(s))c](s) \\ &= [(I-B)R(\cdot, s)c](s), \end{aligned}$$

i.e., $(I-B)R(\cdot, s)c$ is in G_∞ or $R(\cdot, s)c$ is in G for all s in S and c in R^d .

Furthermore, if c_1 and c_2 are d -vectors,

$$\begin{aligned} \langle c_1, R(s,t)c_2 \rangle &= \langle c_1, E(X(s)X'(t))c_2 \rangle \\ &= \langle c_1, E\left(\int_{-r}^s M(s,u)dZ(u)\right)\left(\int_{-r}^t M(t,v)dZ(v)\right)'c_2 \rangle \\ &= \langle c_1, \int_{-r}^s M(s,u)M'(t,u)dk(u)c_2 \rangle \\ &= \int^s \langle M'(s,u)c, M'(t,u)c \rangle dk(u) \end{aligned}$$

$$\begin{aligned}
&= \int_{-r}^{\infty} \langle dE(Z(\cdot)X'(s))c_1, dE(Z(\cdot)X'(t))c_2 \rangle / dk \\
&= Q_{\infty}((I-B)R(\cdot, s)c_1, (I-B)R(\cdot, t)c_2) \\
&= Q(R(\cdot, s)c_1, R(\cdot, t)c_2).
\end{aligned}$$

Let K denote the function from $S \times S$ to the $d \times d$ matrices given by

$$K(s, t) = \begin{cases} k(s+r)I & s \leq t \\ k(t+r)I & t \leq s. \end{cases}$$

Note that K is the reproducing kernel for $\{G_{\infty}, Q_{\infty}\}$.

Theorem. For $s \leq t$, $R(s, t) = [A(P_t A)^* K(\cdot, t)](s)$.

Proof. Since $P_t A$ is a continuous linear transformation of $\{G_{\infty}, Q_{\infty}\}$, we have the adjoint $(P_t A)^*$ exists. Also $AK(\cdot, s)$ is in G for each s in S . Hence, for arbitrary d -vectors c_1 and c_2 and $s \leq t$ in S ,

$$\begin{aligned}
\langle [AK(\cdot, s)](t)c_1, c_2 \rangle &= Q([AK(\cdot, s)](t)c_1, R(\cdot, t)c_2) \\
&= \int_{-r}^t \langle dK(\cdot, s)c_1, d(I-B)R(\cdot, t)c_2 \rangle / dk \\
&= \langle c_1, [(I-B)R(\cdot, t)](s)c_2 \rangle.
\end{aligned}$$

On the other hand,

$$\begin{aligned}
\langle [AK(\cdot, s)](t)c_1, c_2 \rangle &= Q_{\infty}(AK(\cdot, s)c_1, K(\cdot, t)c_2) \\
&= Q_{\infty}(K(\cdot, s)c_1, (P_t A)^* K(\cdot, t)c_2) \\
&= \langle c_1, [(P_t A)^* K(\cdot, t)](s)c_2 \rangle.
\end{aligned}$$

Since c_1 and c_2 are arbitrary, $P_s(I-B)R(\cdot, t) = P_s(P_t A)^* K(\cdot, t)$. But then $P_s A P_s(I-B)R(\cdot, t) = P_s R(\cdot, t) = P_s A(P_t A)^* K(\cdot, t)$ or $R(s, t) = [A(P_t A)^* K(\cdot, t)](s)$.

We might note this can also be written $\langle R(s, t)c_1, c_2 \rangle = Q_{\infty}(K(\cdot, s)c_1, A(P_t A)^* K(\cdot, t)c_2) = Q_{\infty}((P_s A)^* K(\cdot, s)c_1, (P_t A)^* K(\cdot, t)c_2)$. If we introduce the matrix L defined by $L(s, t) = [(P_t A)^* K(\cdot, t)](s) = [P_t(P_t A)^* K(\cdot, t)](s)$, then $L(s, t) = L(t, t)$ when $s \geq t$, $L(\cdot, t)c_1$ is in G_{∞} for $i=1, 2$, and $\langle R(s, t)c_1, c_2 \rangle = Q_{\infty}(L(\cdot, s)c_1, L(\cdot, t)c_2)$.

NUMERICAL METHODS.

For the purpose of illustration assume that everything is scalar valued, i.e., $d = 1$, and $k = I$ the identity on S . Computing $[\text{diag } R](t)$, for a given t , splits into two parts. We are concerned first with function evaluations $L(s, t) = [AK(\cdot, s)](t)$ and then with the integral $Q_\omega(L(\cdot, t), L(\cdot, t)) = \int_{-r}^t |dL(\cdot, t)|^2/dI$. Working backward, observe that the approximating sums $\sum_{p=1}^n |L(s_p, t) - L(s_{p-1}, t)|^2/(s_p - s_{p-1})$ for the integral are nondecreasing with respect to refinement [4].

If $s_p - s_{p-1} = \delta > 0$ for $p = 1, 2, \dots, n$, ϵ is a positive number, and $L(\cdot, t)$ is an approximation for $L(\cdot, t)$ such that $|L(s_p, t) - L(s_p, t)| < \epsilon$ for $p = 1, 2, \dots, n$, then $|\sum_{p=1}^n |L(s_p, t) - L(s_{p-1}, t)|^2/\delta - \sum_{p=1}^n |L(s_p, t) - L(s_{p-1}, t)|^2/\delta| < 4\epsilon[2N_t(L(\cdot, t) + \epsilon)/\delta]$.

Thus, assuming the operator A is given formally by an integro-differential equation, we can compute for a fixed partition $\{s_p\}_{p=0}^n$ of $[-r, t]$ the approximating sum $\sum_s |dL(\cdot, t)|^2/dI$ to any desired accuracy. By refining in turn the partitions s we obtain $[\text{diag } R](t)$.

The method was implemented for the class of scalar systems of the form

$$h(t) = f(t) + \alpha \int_0^t h(u) du + \beta \int_0^t h(u-r_1) du + \gamma \int_0^t h(u-r_2) du.$$

Experiments with $t = 2$, $r_1 = 1/2$, $r_2 = 1$, and various values of α , β , and γ , $-2 \leq \alpha, \beta, \gamma \leq 2$, the algorithm always converged with $\text{err} = 0.001$, i.e., halving the step size for the function evaluation and the integral quadrature produced no change in the first four decimals of $[\text{diag } R](t)$.

CONCLUSIONS

RKH space methods lead to an explicit representation of the covariance kernel for a process governed by an integro-differential equation in terms of solutions that can be obtained by standard numerical methods. Although the presentation of these methods for the sake of clarity was in terms of scalar systems, the methods are readily extended to finite dimensional vector systems. The relative straightforward nature of these methods leads one to anticipate their incorporation into a general methodology for system identification and control design for hereditary systems.

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¹ This research was supported in part by the Air Force Office of Scientific Research Under Grant AFOSR-84-0236

CANONICAL FORMS FOR DECENTRALIZED CONTROL

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Abstract. Geometric, algebraic, and graph theoretic properties of system matrices are presented which allow some autonomy in the choice of component control laws for large scale systems. The results are based upon a decentralized control strategy for two component systems which requires that component control laws be of a specified form, that an additional constraint be placed upon component control requirements, and that component interactions be suitably limited.

*This research was supported in part by the Air Force Office of Scientific Research under Grant AFOSR-84-0236.

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I. INTRODUCTION

Decentralized control problems for finite dimensional state space systems with two independent controllers are considered. Systems of the form

$$\begin{aligned} dh_1/dt &= \alpha_{11} h_1 + \alpha_{12} h_2 + \beta_1 u \\ dh_2/dt &= \alpha_{21} h_1 + \alpha_{22} h_2 + \beta_2 u \end{aligned} \quad (1)$$

and

$$dx/dt = F x + G_1 u_1 + G_2 u_2 \quad (2)$$

where α_{11} , α_{12} , α_{21} , α_{22} , β_1 , β_2 , F , G_1 , G_2 denote matrices of appropriate dimension will be considered. Independent control cannot be achieved without some restriction upon the interaction between system components. The objective of this paper is to characterize geometric and algebraic conditions which allow some autonomy in the choice of controls for systems of the form (1) or (2).

In Section II, a general decentralized control strategy is presented for systems of the form (1). The approach requires that each component control law be the sum of a feedback term and an open loop term, that an additional constraint be added to the component control requirements, and that component interactions be suitably limited. This control strategy is adapted from previous efforts, Reneke (1984), in which variational methods were used to show that some level of autonomous component control is possible when additional constraints are placed upon component requirements and exchanges of related information is allowed. The approach may be referred to as multi-level or hierachial, see Sandell et al. (1978) and Findeisen et al. (1980), in the sense that local models are used in the design of component control laws and a higher level coordination problem is introduced.

The decentralized control strategy to be introduced applies when the interaction between components in (1) is suitably limited, i.e., the components are stably connected. In Section II, this condition is characterized in terms of geometric properties of the matrices α_{11} , α_{12} , α_{21} , and α_{22} .

For systems of the form (2) controllability subspaces, see Wonham (1985 (1), or strong components, see Siljak (1978) or Michel et al. (1978), may be used to decompose control problems into problems for two component systems of the form (1). In Section III such a decomposition is presented and the stably connected condition is characterized in this case. In the final section, interconnected hierarchies of the form (1) are considered. That is α_{11} and α_{22} are assumed to be block lower triangular and a graph theoretic characterization of the stably connected condition is presented.

II. A DECENTRALIZED CONTROL STRATEGY

Let (1) be written in integrated form as

$$h_1 = f_1 + B_1 h_1 + C_2 h_2 + v_1 \quad (1.1)$$

$$h_2 = f_2 + C_1 h_2 + B_2 h_2 + v_2 \quad (1.2)$$

where $B_{ij}h = \alpha_{ij}Ch$, $C_{ij}h = \alpha_{ji}Ch$, $v_i = Cu_i$ with $[Ch](t) = \int_0^t h(\tau) d\tau$ and $f_i(t) = h_i(0)$ for $\{i,j\} = \{1,2\}$. For simplicity, it is assumed that $\beta_i = I_i$ the identity transformation for $i=1,2$. Henceforth whenever indices i or j appear in a statement it is to be assumed that $\{i,j\} = \{1,2\}$. Preliminary calculations show that $[C^2h](t) = \int_0^t (t-\tau)h(\tau) d\tau$ and, using Laplace transforms that $[(I-C^2)^{-1}h](t) = h(t) + \int_0^t \sinh(t-\tau)h(\tau) d\tau$. Of course the underlying function

space must be restricted so that this inverse operator exists. Similarly, if $[A_i v](t) = \int_0^t \exp(\alpha_{ii}(t-\tau))v(\tau)d\tau$ then $A_i = (I-B_i)^{-1}$ and $A_i^{-1} = I-B_i$.

The control problem is to choose v_1, v_2 independently to steer the components from a given position $h_i(0) = h_{i0}$ to a desired terminal position $h_i(T) = h_{iT}$ for $i=1,2$.

Our decentralized control strategy, which allows some autonomy in the choice of component controls, requires that for a given parameter c_i the constraint

$$[(I-C^2)^{-1}A_j C_i h_i](T) = c_i \quad (3.1)$$

be added to the i th control requirements, that component interactions be limited so that

$$(I-C^2)^{-1}A_j C_i (I-C^2)^{-1}A_i C_j = 0, \quad (4.1)$$

and that the control law for the i th component be of the form

$$v_i(t) = [A_i^{-1}C^2(h_i - Y_i) + A_i^{-1}\{k_{i1}(t, \lambda_i) + k_{i2}(t, v_i)\}](t) \quad (5.1)$$

where Y_i denotes a desired trajectory for the response h_i , the functions

$k_{i1}(t, \lambda)$ and $k_{i2}(t, v)$ are linear in the parameters λ_i, v_i respectively, and λ_i, v_i satisfy

$$h_{iT} = [(I-C^2)^{-1}\{A_i f_i + c_j - C^2 Y_i + k_{i1}(t, \lambda) + k_{i2}(t, v)\}](T) \quad (6.1)$$

$$c_i = [D_i(I-C^2)^{-1}\{A_i f_i - C^2 Y_i + k_{i1}(t, \lambda) + k_{i2}(t, v)\}](T)$$

with $D_i = (I-C^2)^{-1}A_j C_i$.

THEOREM 1. If the control law (5.1) is used in component (1.1) and condition (4.1) holds then the response h_i satisfies $h_i(0) = h_{i0}$, $h_i(T) = h_{iT}$, and the additional constraint $[(I-C^2)^{-1}A_j C_i h_i](T) = c_i$ is satisfied.

Since $[C_j h](0) = 0$, $[B_i h](0) = 0$, and $[A_i^{-1} h](0) = h(0)$ for all functions h , it follows that $h_i(0) = f_i(0) = h_{i0}$. Let $w_i(t) = [A_i^{-1} \{k_{i1}(\cdot, \lambda_i) + k_{i2}(\cdot, v_i)\}](t)$. Substituting (5.1) into (1.1), one obtains $(I - B_i)(I - C^2)h_i = f_i + C_j h_j - A_i^{-1} C^2 Y_i + w_i$. Thus $h_i = (I - C^2)^{-1} \{A_i f_i + A_i C_j h_j - C^2 Y_i + k_{i1}(\cdot, \lambda_i) + k_{i2}(\cdot, v_i)\}$. It follows that $h_i(T) = h_{iT}$ and $[(I - C^2)^{-1} A_j C_i h_i](T) = c_i$ since λ_i, v_i satisfy equations (6.1).

In this decentralized control strategy the controller for the i^{th} component is free to choose the desired response Y_i and the functions k_{i1} , k_{i2} so that condition (3.1) is satisfied and equations (6.1) can be solved for λ_i, v_i . The condition (3.j) is the only information about the j^{th} component used to determine the control law for the i^{th} component. Here component controls are coordinated in one step, i.e., for given c_1, c_2 component control objectives are achieved when controls are determined by (5.1), (5.2) and the constraints (3.1), (3.2) are satisfied. Other decentralized control strategies typically require off line iterative calculations between controllers for coordination. In our strategy, choice of appropriate values for the parameters c_1, c_2 leads to a higher level coordination problem.

Optimization techniques can be used to determine specific component control laws. For example, Reneke (1984) shows that a control law of form (5.1) is optimal if the performance index $J(u_i, h_i) = (1/2) \int_0^T [h_i]^2(\tau) + [A_i h_i]^2(\tau) d\tau$ is minimized subject to the constraints $h_i = f_i + B_i h_i + C_j h_j + v_i$, $h_i(T) = h_{iT}$, $[D_i h](T) = c_i$ given that $[D_j h_j](T) = c_j$ and $D_i D_j = 0$. The standard quadratic cost functional $(1/2) \int_0^T [v_i]^2(\tau) + [h_i]^2(\tau) d\tau$ can also be used as a basis for the choice of component control laws, see Fennell et al. (1985) for an example.

Condition (4.1), (4.2), i.e., $D_1 D_2 = D_2 D_1 = 0$, represent a restriction upon the interaction between the two components. Two components of the form (1) satisfying such conditions are referred to as stably connected components. The objective of this paper is to present geometric and algebraic characterizations of the stably connected condition.

THEOREM 2. The components in (1) are stably connected if and only if $C_1 A_1 C_2 = 0$ and $C_2 A_2 C_1 = 0$.

If $h = (I - C^2)^{-1} f$ then $h = f + C^2 h$ and $C_i h = C_i f + C_i C^2 h = C_i f + C^2 C_i h$. By uniqueness $C_i h = (I - C^2)^{-1} C_i f$, i.e. $C_i (I - C^2) = (I - C^2) C_i$. Therefore $D_1 D_2 = D_2 D_1 = 0$ if and only if $C_1 A_1 C_2 = 0$ and $C_2 A_2 C_1 = 0$.

Thus although each component can affect the other the stably connected condition implies there is no feed-through effect. Basic properties of finite dimensional systems yield the following geometric characterization of the stably connected condition.

THEOREM 3. The system (1) is stably connected if and only if

$$\text{span}[\alpha_{12}, \alpha_{11}\alpha_{12}, \alpha_{11}^2\alpha_{12}, \dots] \subset \text{Ker}(\alpha_{21}) \text{ and}$$

$$\text{span}[\alpha_{21}, \alpha_{22}\alpha_{21}, \alpha_{22}^2\alpha_{21}, \dots] \subset \text{Ker}(\alpha_{12}).$$

Let $y(t) = [A_1 C_2 v](t)$ for some v , $0 \leq t \leq T$, then invariance properties of controllability subspaces imply $y(t)$ belongs to $\text{span}[\alpha_{12}, \alpha_{11}\alpha_{12}, \alpha_{11}^2\alpha_{12}, \dots]$ for $0 \leq t \leq T$. If $\text{span}[\alpha_{12}, \alpha_{11}\alpha_{12}, \alpha_{11}^2\alpha_{12}, \dots] \subset \text{Ker}(\alpha_{21})$ then $C_1 y = 0$ and $C_1 A_1 C_2 = 0$. Similarly $C_2 A_2 C_1 = 0$. For the converse, let w be in $\text{span}[\alpha_{12}, \alpha_{11}\alpha_{12}, \alpha_{11}^2\alpha_{12}, \dots]$ and choose v so that $w = [A_1 C_2 v](T)$. If $C_1 A_1 C_2 = 0$ then $C_1 y = 0$ with $y = A_1 C_2 v$. It follows that $\alpha_{21} y(t) = 0$ for $0 \leq t \leq T$ and hence w belongs to $\text{Ker}(\alpha_{21})$.

For example if

$$\alpha_{11} = \alpha_{22} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \text{ and } \alpha_{12} = \alpha_{21} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$$

then the system (1) is stably connected.

This characterization of stably connected components is similar to geometric conditions that arise in disturbance decoupling problems, see Wonham (1985). In fact, disturbance decoupling methods can be used to modify certain systems of the form (1) through local feedback so that the resulting two components system is stably connected.

Let $A: \mathbb{R}^n \rightarrow \mathbb{R}^n$, $B: \mathbb{R}^m \rightarrow \mathbb{R}^n$ be linear transformations. With notation as in Wonham (1985), a subspace V of \mathbb{R}^n is called A-B invariant if there is a linear transformation $F: \mathbb{R}^n \rightarrow \mathbb{R}^m$ such that $(A+BF)V \subset V$. For a subspace K of \mathbb{R}^n , $T(A,B,K)$ is the set of all A-B invariant subspaces which are also subspaces of K . The notation $V^* = \sup T(A,B,K)$ denotes the element of $T(A,B,K)$ which contains all other elements of $T(A,B,K)$. The following remark is a restatement of the solution to the disturbance decoupling problem.

Remark. If $\text{Im}(\alpha_{12}) \subset \sup(\alpha_{11}, \beta_1, \text{Ker}(\alpha_{21}))$ and $\text{Im}(\alpha_{21}) \subset \sup(\alpha_{22}, \beta_2, \text{Ker}(\alpha_{12}))$ then there exist matrices F_1, F_2 such that the two component system

$$dh_1/dt = (\alpha_{11} + \beta_1 F_1)h_1 + \alpha_{12}h_2 + \beta_1 u_1$$

$$dh_2/dt = \alpha_{21}h_2 + (\alpha_{22} + \beta_2 F_2)h_2 + \beta_2 u_2$$

is stably connected.

Use of high gain, local feedback loops to reject interconnections and to aid in the synthesis of decentralized control laws has been reported in the work of Young (1983), (1985).

III. DECOMPOSITION

For the system (2) consider the problem of choosing controls u_1, u_2 , independently, which steer the system from a given initial position to a desired terminal position. In this section a method to decompose the control problem for (2) into a problem for a two component system of the form (1) is presented. The approach uses standard methods to decompose systems of the form (2) into block triangular form. Decomposition of (2) into block triangular form using controllability subspaces or strong components provides a geometric view of control coordination possibilities.

Let $F = (F_{ij})$, $G_1 = (G_{1ij})$ and $G_2 = (G_{2ij})$ denote $n \times n$, $n \times m$, $n \times r$ matrices with respect to the standard basis for $X = \mathbb{R}^n$ and let the components of x, u_1, u_2 be denoted by $x = \text{col}(x_1, \dots, x_n)$, $u_1 = \text{col}(u_{11}, \dots, u_{1m})$, and $u_2 = \text{col}(u_{21}, \dots, u_{2r})$.

Controllable Canonical Form. A coordinatization of the basic controllable canonical form, introduced by Kalman, Ho, and Narendra (1961) will be presented, see also Wonham (1985).

Assuming (2) is controllable, let \tilde{X}_i denote the controllability subspace determined by the pair (F, G_i) $i=1,2$. Controllability implies $X = \tilde{X}_1 + \tilde{X}_2$. If $\tilde{X}_3 = \tilde{X}_1 \cap \tilde{X}_2$ then either $\tilde{X}_3 = \{0\}$, $\tilde{X}_3 = \tilde{X}_1$, $\tilde{X}_3 = \tilde{X}_2$, or \tilde{X}_3 is a proper nontrivial subspace \tilde{X}_1 and \tilde{X}_2 . In any case there exist subspaces X_1 and X_2 such that $\tilde{X}_1 = X_1 \oplus X_3$, $\tilde{X}_2 = X_2 \oplus X_3$ and $X = X_1 \oplus X_2 \oplus X_3$.

With respect to the standard basis, let W_i denote a matrix whose columns form a basis for X_i , $i=1,2,3$ and let P be the $n \times n$ matrix defined by $P = [W_1, W_2, W_3]$. Since X_1, X_2 , and X_3 are A invariant the change of coordinates $x = Ph$ leads to the controllable canonical form

$$\dot{h}/dt = ah + \beta_1 u_1 + \beta_2 u_2 \quad (7)$$

where

$$\alpha = \begin{bmatrix} A_{11} & 0 & 0 \\ 0 & A_{22} & 0 \\ A_{31} & A_{32} & A_{33} \end{bmatrix} \quad \beta_1 = \begin{bmatrix} \beta_{11} \\ 0 \\ \beta_{13} \end{bmatrix} \quad \beta_2 = \begin{bmatrix} 0 \\ \beta_{22} \\ \beta_{23} \end{bmatrix}$$

with $\alpha = P^{-1}FP$, $\beta_1 = P^{-1}G_1$ and $\beta_2 = P^{-1}G_2$.

Here the subsystems determined by A_{11} , β_{11} and A_{22} , β_{22} are controllable whereas the system determined by the pair A_{33} , (β_{13}, β_{23}) need not be controllable, see Wonham (1985), as the example

$$\alpha = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}, \quad \beta_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad \beta_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

illustrates.

Numerical computation of controllability subspaces using recent advances in numerical linear algebra is considered in the work of Klema and Laub (1980), Van Dooren (1981), Boley (1981).

Block Triangular Form. Transformation of (2) into block triangular form can be achieved by graph theoretic methods, see Siljak (1978), Pichai, et. al. (1983) or Michel et. al. (1978). Let $D = \{V, E\}$ be the directed graph with vertex set $V = S \cup C_1 \cup C_2$ where $S = \{x_1, \dots, x_n\}$, $C_1 = \{u_{11}, \dots, u_{1m}\}$, $C_2 = \{u_{21}, \dots, u_{2r}\}$ and edge set $E = \{(x_i, x_j): a_{ji} \neq 0\} \cup \{(u_{1i}, x_j): G_{1ji} \neq 0\} \cup \{(u_{2i}, x_j): G_{2ji} \neq 0\}$. A subgraph $D = \{V, E\}$ of D is called a strong component provided for each pair of vertices x_i, x_j in V there is a directed path from x_i to x_j with each edge in E and V is maximal with respect to this property. Let $D_i = \{V_i, E_i\}$, $i=1, \dots, m$ denote the strong components of D . Clearly $V_i \cap V_j = \emptyset$, $E_i \cap E_j = \emptyset$ for $i \neq j$ and $V = \bigcup_{i=1}^m V_i$. The condensation graph $CD = \{CV, CE\}$ has m vertices, one associated with each strong component, say $v_i - D_i$, and an edge from v_i to v_j if and only if there is a u in V_i and a v in V_j such

that (u,v) is in E . The condensation graph determines a relabeling of indices in (2) so that the resulting system matrix is block lower triangular. Hence, the system is decomposed into a "hierachy of m interconnected components". Let $S_1 = \{D_k: \text{for some } i \text{ there is a path in the condensation graph from } u_{1i} \text{ to } D_k\}$. Similary define S_2 . Let $S_3 = S_1 \cup S_2$ and $S_1 = S_1 - S_3, S_2 = S_2 - S_3$. Then S_1, S_2, S_3 determine a relabeling of vertices so that (2) is of the form (7) with

$$\alpha = \begin{bmatrix} A_{11} & 0 & 0 \\ 0 & A_{22} & 0 \\ A_{31} & A_{32} & A_{33} \end{bmatrix} \quad \beta_1 = \begin{bmatrix} \beta_{11} \\ 0 \\ \beta_{13} \end{bmatrix} \quad \beta_2 = \begin{bmatrix} 0 \\ \beta_{22} \\ \beta_{23} \end{bmatrix}.$$

Control Problem Decomposition. Controls u_1 and u_2 steer the solution of (2) from $x(0) = x_0$ to $x(T) = x_T$ if and only if u_1, u_2 steer the solution of (7) from $h(0) = h_0$ to $h(T) = h_T$ where $h_0 = Px_0, h_T = Px_T$ if (7) arises from the controllable canonical form or the components of h_0, h_T are obtained from those of x_0, x_T by using the permutation of indices which determines the block lower triangular form. To further decompose the problem let $0 \leq m_0, m_1, m_2 \leq 1$,

$$\begin{aligned} h_{10} &= \begin{bmatrix} h_{01} \\ 0 \\ (1-m_0)h_{03} \end{bmatrix} & h_{1T} &= \begin{bmatrix} h_{T1} \\ 0 \\ (1-m_0)h_{T3} \end{bmatrix} \\ h_{20} &= \begin{bmatrix} 0 \\ h_{02} \\ m_0h_{03} \end{bmatrix} & h_{2T} &= \begin{bmatrix} 0 \\ h_{T2} \\ m_0h_{T3} \end{bmatrix} \end{aligned}$$

and consider the following two control problems.

CONTROL PROBLEM I. Find u_1 which steers the solution of

$$dh_1/dt = \begin{bmatrix} A_{11} & 0 & 0 \\ 0 & 0 & 0 \\ (1-m_1)A_{31} & 0 & (1-m_1)A_{33} \end{bmatrix} h_1 + m_2 \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & A_{32} & A_{33} \end{bmatrix} h_2 + \begin{bmatrix} \beta_1 \\ 0 \\ \beta_3 \end{bmatrix} u_1 \quad (7.1)$$

from $h_1(0) = h_{10}$ to $h_1(T) = h_{1T}$.

CONTROL PROBLEM II. Find u_2 which steers the solution of

$$dh_2/dt = m_1 \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ A_{31} & 0 & A_{33} \end{bmatrix} h_1 + \begin{bmatrix} 0 & 0 & 0 \\ 0 & A_{22} & 0 \\ 0 & (1-m_2)A_{32} & (1-m_2)A_{33} \end{bmatrix} h_2 + \begin{bmatrix} 0 \\ \beta_2 \\ \beta_3 \end{bmatrix} u_2 \quad (7.2)$$

from $h_2(0) = h_{20}$ to $h_2(T) = h_{2T}$.

Here (7.1) and (7.2) are of the form (1). Let u_1 and u_2 solve CONTROL PROBLEMS I, II respectively. Notice that the second component of h_1 and the first component of h_2 are always zero. It follows that $h = h_1 + h_2$ satisfies (7) and $h(0) = h_0$, $h(T) = h_T$. That is u_1 and u_2 steer the solution of (7) from $h(0) = h_0$ to $h(T) = h_T$.

Clearly if $m_1=0$ or $m_2=0$ then the components in (7.1), (7.2) are stably connected.

THEOREM 4. The two components of (7.1), (7.2) with m_1 and m_2 non-zero are stably connected if and only if $A_{33}A_{31} = 0$, $A_{33}A_{32} = 0$, and $A_{33}^2 = 0$.

With notation as in (7.1), (7.2) the controllability subspace determined by $(\alpha_{11}, \alpha_{12})$ is spanned by the columns of the matrix

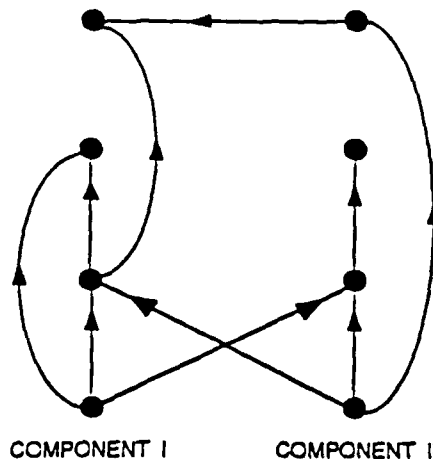
$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & 0 & 0 & \cdot & \cdot & \cdot \\ A_{32} & A_{33} & A_{33}A_{32} & A_{33}^2 & A_{33}^2A_{32} & \cdot & \cdot & \cdot \end{bmatrix} \cdot$$

A vector $x = \text{col}(x_1, x_2, x_3)$ is in the $\text{Ker}(\alpha_{21})$ if and only if $A_{31}x_1 + A_{33}x_3 = 0$. Clearly the columns of the above matrix lie in the $\text{Ker}(\alpha_{21})$ if and only if $A_{33}A_{32} = 0$ and $A_{33}^2 = 0$. Similarly, the controllability subspace determined by $(\alpha_{22}, \alpha_{21})$ lies in the $\text{Ker}(\alpha_{12})$ if and only if $A_{33}A_{31} = 0$ and $A_{33}^2 = 0$. The result follows from Theorem 2.

With notation as in (1). Theorem 4 implies that the components in (7.1), (7.2) are stably connected if and only if $\alpha_{21}\alpha_{12} = 0$ and $\alpha_{12}\alpha_{21} = 0$.

IV. INTERCONNECTED HIERARCHIES.

Graphic theoretic conditions for (1) to be stably connected are presented in the case α_{11} and α_{22} have a hierarchial structure. Assume $\alpha_{11} = (\alpha_{11,ij})_{i,j=1,\dots,m}$ and $\alpha_{22} = (\alpha_{22,ij})_{i,j=1,\dots,l}$ with $\alpha_{11,ij}=0$ and $\alpha_{22,ij}=0$ for $i>j$. Let $\alpha_{12} = (\alpha_{12,ij})$ and $\alpha_{21} = (\alpha_{21,ij})$ denote $m \times n$, $n \times m$ matrices, respectively. With notation as in Section III the system directed graph $D=(V,E)$ may be described as follows $V = V_1 \cup V_2 = \{x_{11}, \dots, x_{1m}\} \cup \{x_{21}, \dots, x_{2l}\}$ and $E = \{(x_{1i}, x_{1j}): \alpha_{11,ji} \neq 0\} \cup \{(x_{1i}, x_{2j}): \alpha_{21,ji} \neq 0\} \cup \{(x_{2i}, x_{1j}): \alpha_{12,ji} \neq 0\} \cup \{(x_{2i}, x_{2j}): \alpha_{22,ji} \neq 0\}$. Such a directed graph may be depicted as follows:



THEOREM 5. With $D=(V,E)$ as defined above, the components in (1) are stably connected if and only if there exist no directed paths with initial and terminal nodes in V_1 which transverse a node in V_j for $\{i,j\} = \{1,2\}$.

To see that $\text{span}[\alpha_{12}, \alpha_{12}\alpha_{12}, \alpha_{11}^2\alpha_{11}, \dots] \subset \text{Ker}(\alpha_{21})$ we argue that $\alpha_{21}\alpha_{11}^k\alpha_{12} = 0$ for $k=0, 1, \dots, m-1$. The product $\alpha_{21}\alpha_{12} = 0$ since there are no directed paths of length two with initial and terminal nodes in V_2 which transverse a node in V_1 . The product $\alpha_{21}\alpha_{11}\alpha_{12} = 0$ since there are no directed paths of length three with initial and terminal nodes in V_2 which transverse two nodes in V_1 . Similarly $\alpha_{21}\alpha_{11}^k\alpha_{12} = 0$ for $k=0, 1, \dots, m-1$.

Theorem 4 generalizes to the case where $D_1 = (V_1, E_1)$ and $D_2 = (V_2, E_2)$ represent the condensation graphs determined by α_{11} and α_{22} respectively. Here α_{11}, α_{22} are block triangular, and α_{12}, α_{21} are partitioned accordingly.

CONCLUDING REMARK

A large scale system with multiple components is said to be stably connected provided each pair of components satisfy the stably connected condition. The decentralized control strategy presented in this paper can be generalized to a decentralized control strategy for stably connected systems. If a system matrix is block triangular then the blocks determine components such that the system is stably connected. On the other hand, one may view the components as basic building blocks and the results of this paper indicate how to connect the components together to form more complex systems while at the same time maintaining some autonomy in the choice of component control laws.

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